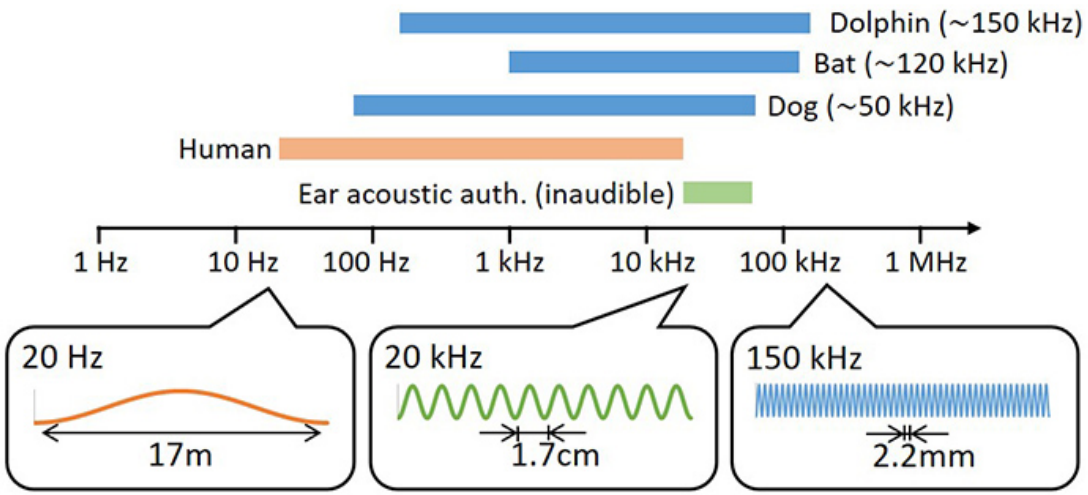
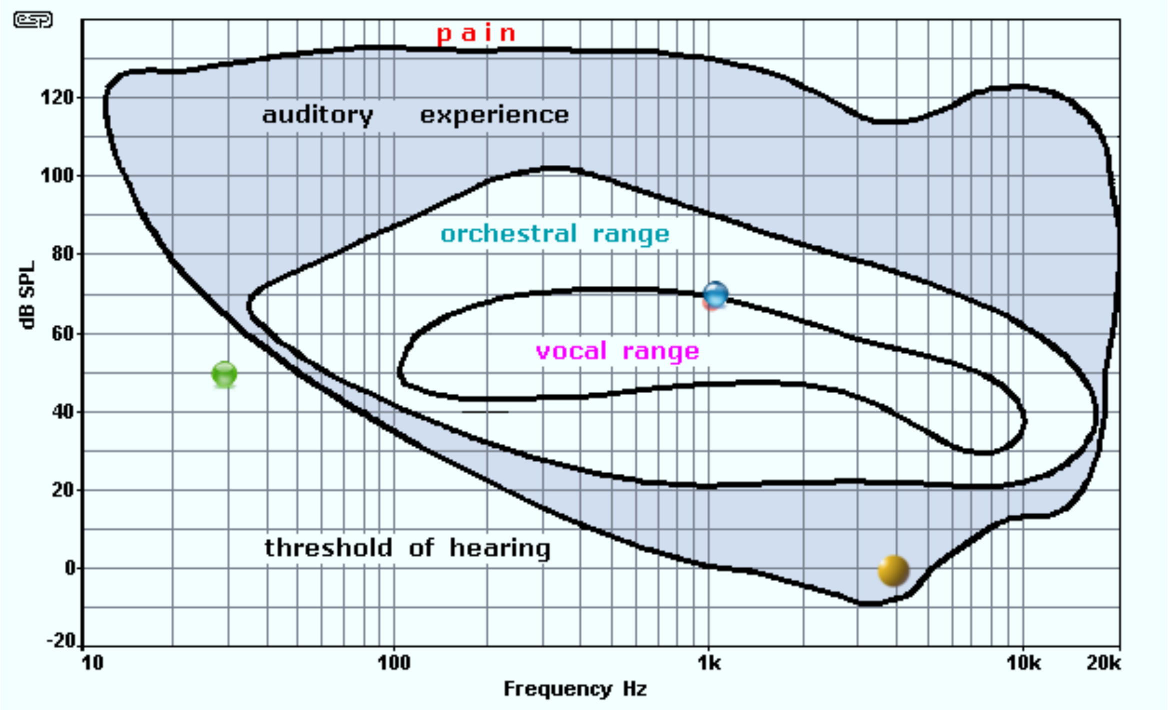
Modern Physics – Topics in Medical Physics

Ultrasound

- A region of sound spectrum > 20 KHz. Infrasound refers to < 20 Hz. Humans can hear from 20 to 20 kHz. Typically 15 kHz which has wavelength of 0.1 m in water.
- Medically, used as a diagnostic tool complementing x-rays, nuclear medicine and magnetic resonance.
    - Pros: real time, low cost, portable equipment. Can provide
    - Cons: image quality not as other techniques (depending on what is being imaged)
- Described mathematically with the wave equation.



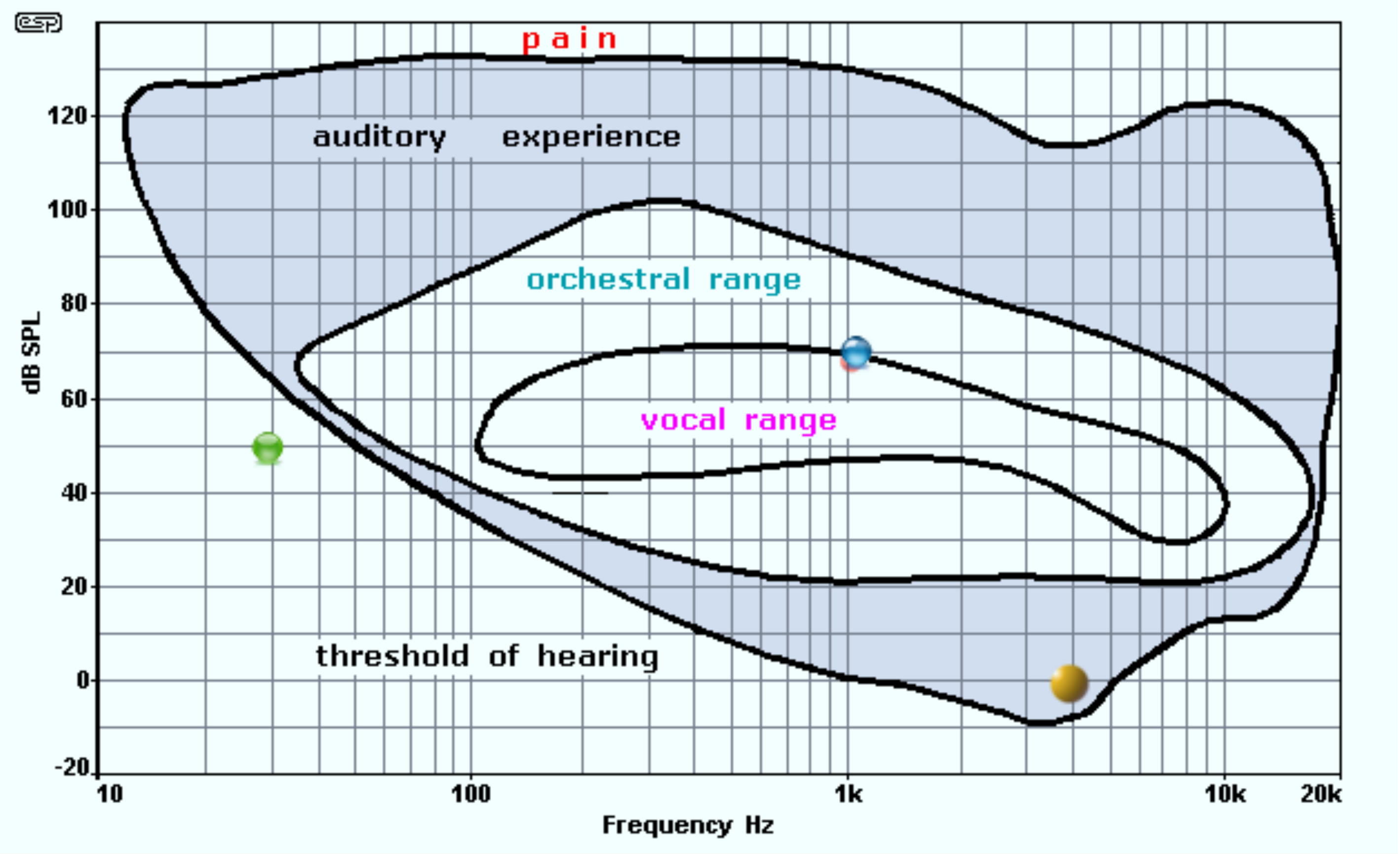


http://www.maineghosthunters.org/blogs/2011/05/24/evps-frequencies-and-software/

NEC https://www.nec.com/en/press/201802/global_20180227_03.html

Human Hearing

- The decibel (dB) is a measure of the sound pressure (measured in Pascals).
- Sounds louder than 120 dB typically begin to cause pain and damage to ear.
- Sound at two close frequencies are often perceived as averages by the ear. At 10 Hz difference a "beat" effect experienced instead.



http://www.maineghosthunters.org/blogs/2011/05/24/evps-frequencies-and-software/

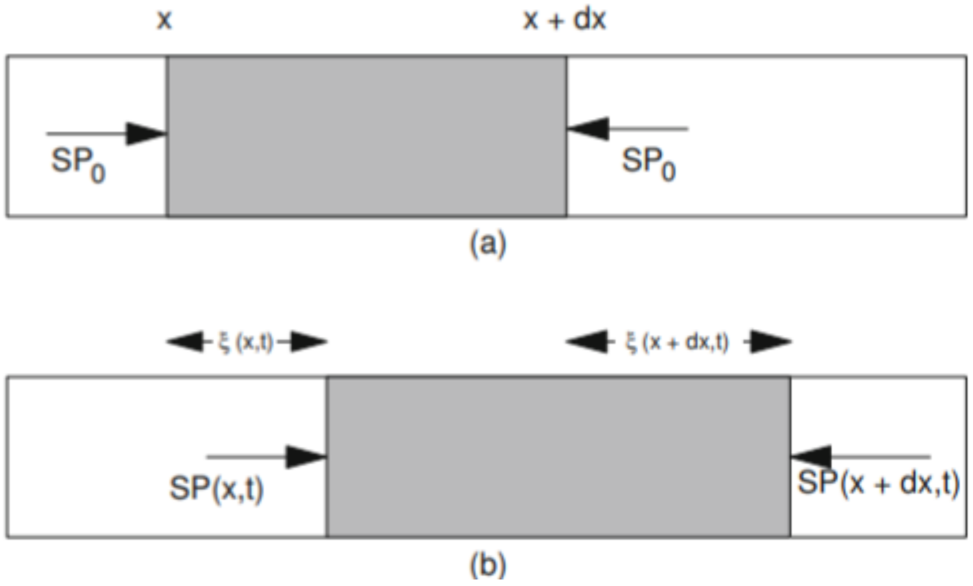
Ultrasound

Species	Range (Hz)
Turtle	20–1,000
Goldfish	100–2,000
Frog	100–3,000
Pigeon	200–10,000
Sparrow	250–12,000
Human	20–20,000
Chimpanzee	100-20,000
Rabbit	300–45,000
Dog	50–46,000
Cat	30–50,000
Guinea pig	150–50,000
Rat	1,000–60,000
Mouse	1,000–100,000
Bat	3,000–120,000
Dolphin (*Tursiops*)	1,000–130,000

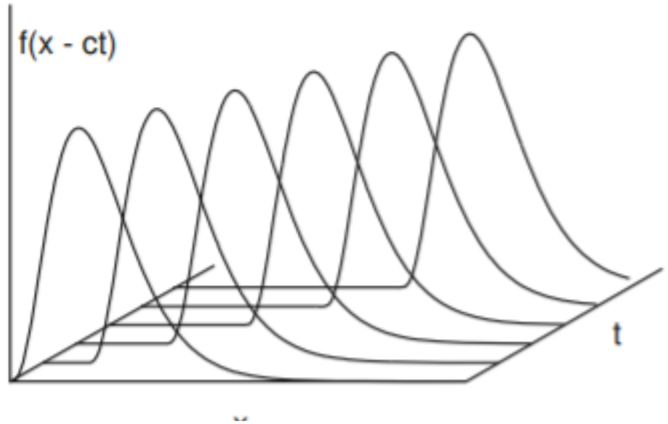
While not completely verified, this chart provides additional scientific support for why cats and dogs are man's best choice for animal friends.

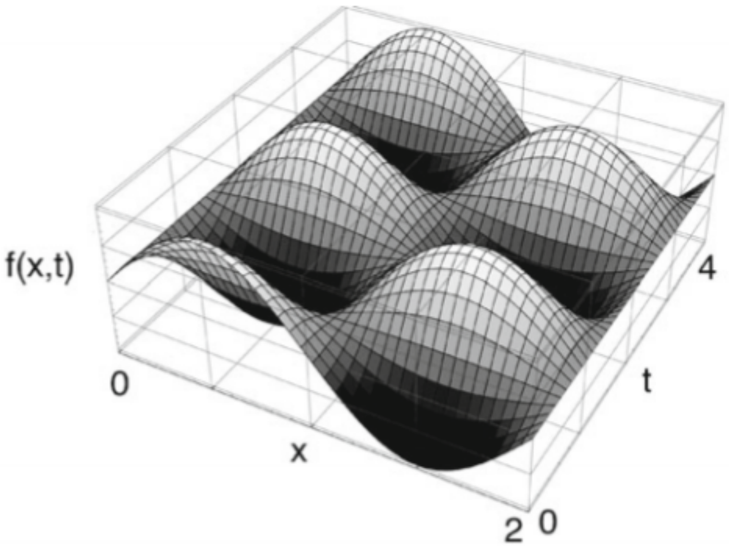
http://donrathjr.com/audible-range-human-hearing/

Plane Waves in an Elastic Rod



Properties of the Wave Equation





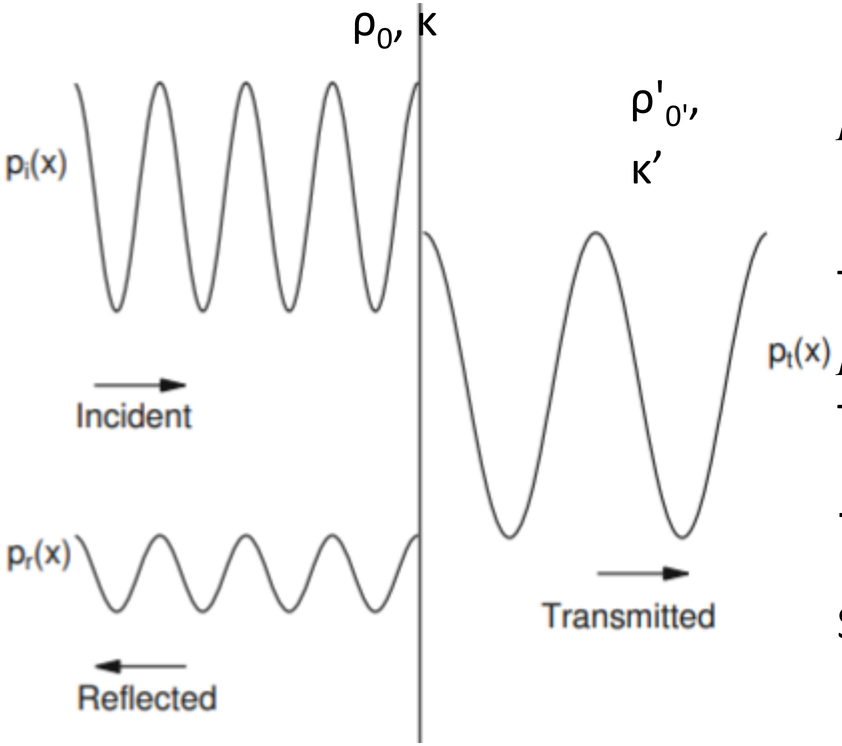
$$p(x,t) = pcos(\omega t)\sin(kx) with\ k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

- Speed of sound in water ~ 1400 m/s
- Speed of sound in air ~ 344 m/s
- Speed of sound in tissue ~ 1540 m/s

- Waves → traveling waves, standing waves

Reflection and transmission of sound at a boundary

$$p_i(x,t) = p_i sin\left[\frac{\omega}{c_1}(x - c_1 t)\right],\; p_r(x,t) = p_r sin\left[\frac{\omega}{c_1}(x + c_1 t)\right]$$



$$p_t(x,t) = p_t sin\left[\frac{\omega}{c_2}(x - c_2 t)\right]$$

The amplitude of pressure in each fluid 1 and fluid 2 is equal at the boundary.

$$p_i + p_r = p_t$$

The velocity amplitude matching is given by

$$\frac{p_i - p_r}{Z_1} = \frac{p_t}{Z_2}$$

Solving the system of two equations gives

$$p_r = \frac{Z_2 - Z_1}{Z_2 + Z_1} p_i \; and \; p_t = \frac{2Z_2}{Z_2 + Z_1} p_i$$

Impedances ($Z = \rho_0 c = \sqrt{(\rho_0/\kappa)}$) $Z_2 = 2\,Z_1$

The intensity of the sound wave in W/m2 is calculated as

$$R = \frac{I_r}{I_i} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 \quad T = \frac{I_t}{I_i} = \frac{4Z_1 Z_2}{(Z_1 + Z_2)^2}$$

$$I(t) = p(t)v(t) = p_0 v_0 \sin^2 \omega t \quad I = \frac{1}{2} p_0 v_0 = \frac{\frac{1}{2} p_0^2}{Z}$$

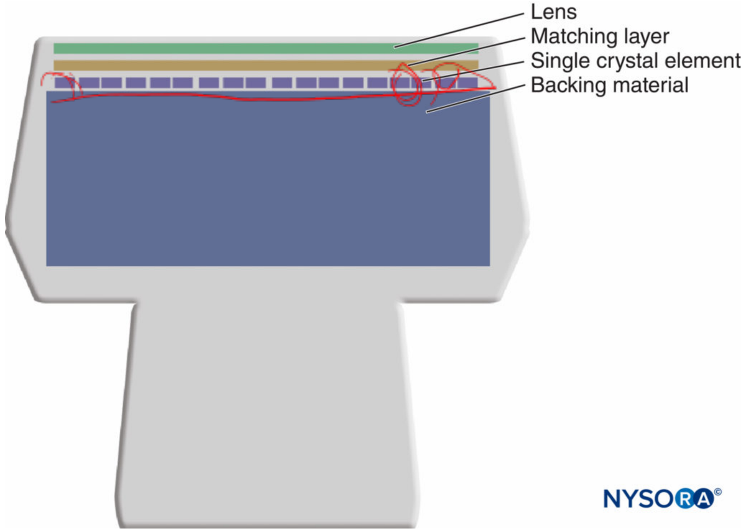
Ultrasound of Human Body

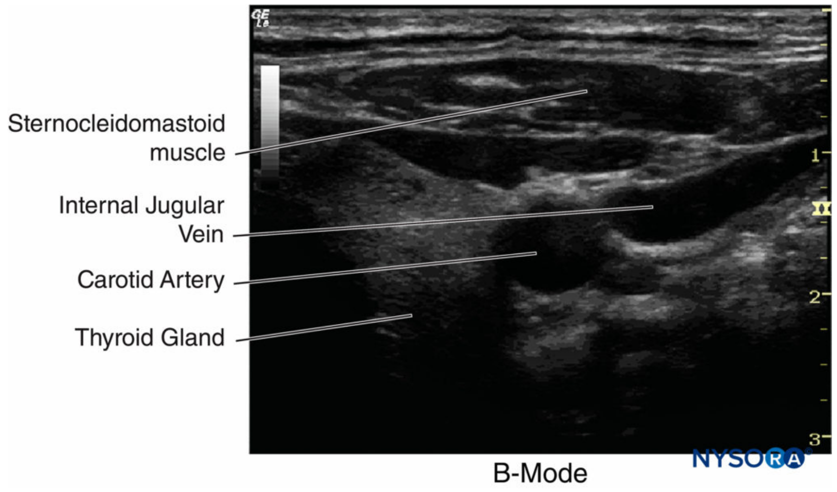
Skin intensities in diagnostic ultrasound are 0.1 (0.08 to 0.5) W/m^2 for an obstetric exam to larger for heart or blood vessel imaging. The effect is rising temperature of tissue from sound wave.

- For example, fetal ultrasounds are routinely conducted during pregnancy to monitor fetal growth and health.
- The image is a *B scan* of a 16 week fetus.

Ultrasound of Human Body: Imaging With Ultrasound Transducers

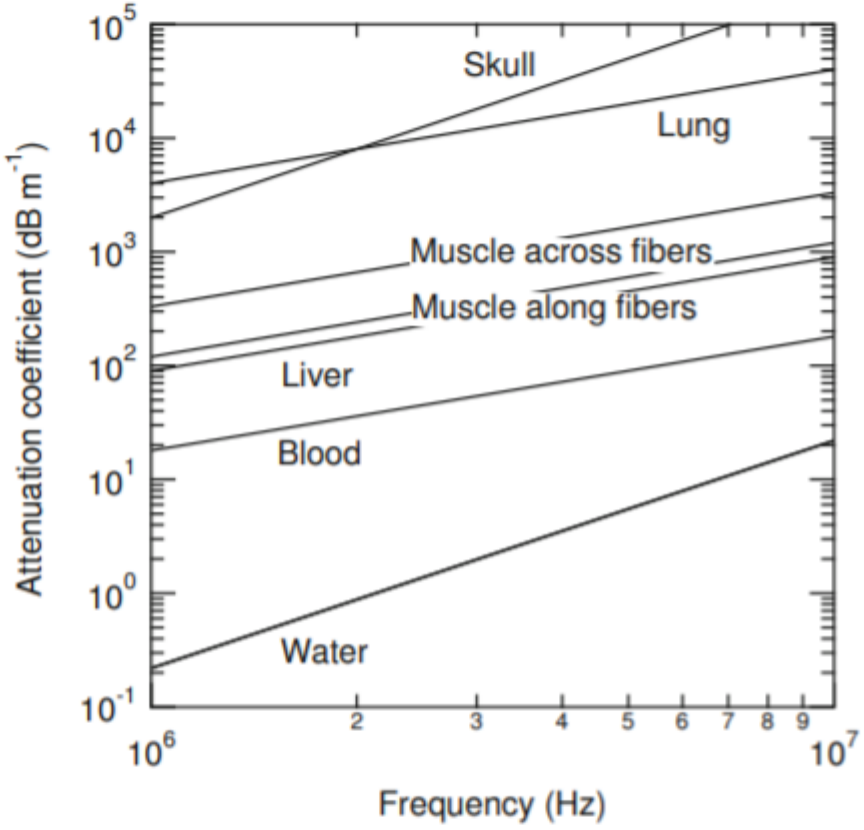
- Produced using piezoelectric transducer.
- A piezoelectric material converts stress (or pressure) into an electric field, and vice versa.
- Thus, a high-frequency oscillating voltage applied across a piezoelectric material creates a sound wave of the same frequency and conversely.
- Lead zirconate titanate (PZT) is a well known medical transducer with density 7500 kg m^{-3}, speed of sound 4065 m/s, acoustic impedance 30 Mpa s/m (about half of the electrical energy is converted to sound energy).





https://www.nysora.com/foundations-of-regional-anes equipment/physics-of-ultrasound/

Ultrasound Attenuation



- Attenuation coefficients for ultrasound of body tissue

$$\alpha = -\frac{1}{p}\frac{dp}{dx}$$

$$p(x) = p(0)e^{-\alpha x} \; and \; from \; this \; I(x) = I(0)e^{-2\alpha x}$$

- M=2α. Expressed attenuation in dB/m.

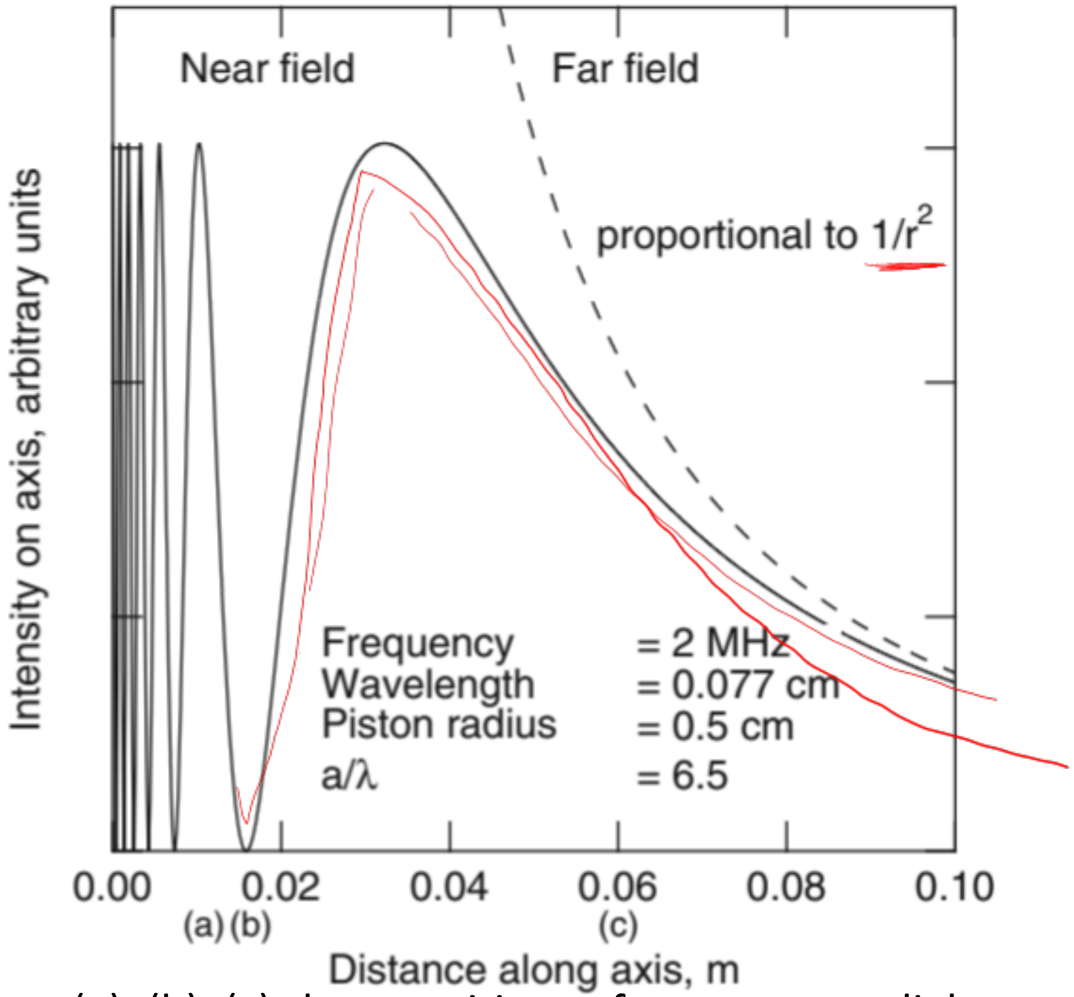
Air and water both attenuate sound. Water actually transmits sound better than air, although attenuation depends strongly on frequency and salt content. In fresh water 1 kHz attenuates by 4e-4 dB/km. In salt water 10 times higher.

Ultrasound Attenuation

Sound	Intensity ($W\ m^{-2}$)	Level (dB, A weighting)
Rocket launch pad	10^5	170
	10^4	160
	10^3	150
	10^2	140
F-84 jet at takeoff, 25 m from the tail; Large pneumatic riveting machine (1 m); Boiler shop (maximum level); Peak sound level at a rock concert	10	130
Sound that produces pain	1	120
Woodworking shop	10^{-1}	110
Near a pneumatic drill ("jack hammer")	10^{-2}	100
Inside a motor bus	10^{-3}	90
Urban dwelling near heavy traffic	10^{-4}	80
Busy street	10^{-5}	70
Speech at 1 m	10^{-6}	60
Office	10^{-7}	50
Average dwelling	10^{-8}	40
Maximum background sound level tolerable in a broadcast studio	10^{-9}	30
Whisper; maximum background sound level tolerable in a motion picture studio	10^{-10}	20
	10^{-11}	10
Minimum perceptible sound	10^{-12}	0

Ultrasound Transducers

- The intensity of sound radiated from a transducer of radius a.
- The entire piston and surrounding fluid vibrates with velocity $\frac{d\xi}{dt} = v_0 cos\omega t$
- The amplitude of spherical wave decreases as 1/r and intensity falls as the square.
- At time t the phase of the wave is kept as the phase leaving the annular ring r'dr' at earlier time t-r/c.



(a), (b), (c) show positions of transverse radial scans

$$p \sim \frac{d\xi(z,t)}{dt} = \int_0^a 2\pi r' dr' \frac{\cos\left[\omega(t - \frac{r}{c})\right]}{r}$$

$$p \sim 2\pi \int_{r=z}^{r=\sqrt{a^2+z^2}} rdr \frac{cos\left[\omega(t - \frac{r}{c})\right]}{r} = \frac{2\pi}{k}\left[\sin\left[\omega\left(t - \frac{1}{c}\sqrt{(a^2+z^2}\right)\right] - sin\left[\omega(t - \frac{z}{c})\right]\right]$$

$$I \sim \sin^2\left[\frac{\omega}{2c} z - \sqrt{(a^2+z^2)}\right]$$

In the far field the intensity goes as the inverse r squared.

Ultrasound Transducers

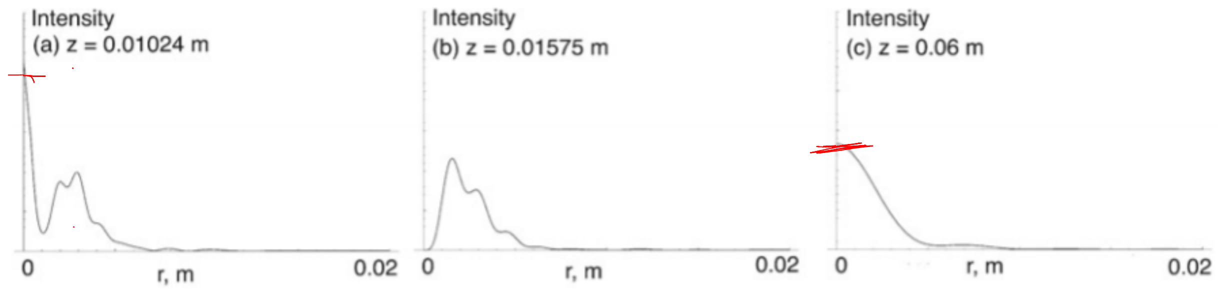


Fig. 13.15 Scans across the beam from the transducer shown in Fig. 13.14. **a** In the near field at an on-axis maximum 0.01024 m from the transducer. **b** In the near field at an on-axis minimum 0.01575 m from the transducer. **c** In the far field 0.060 m from the transducer

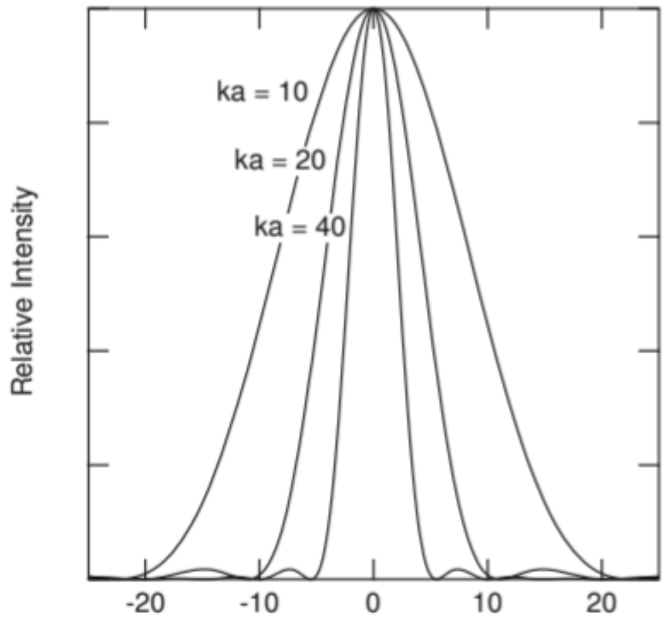


Fig. 13.16 The far-field intensity as a function of angle, calculated from Eq. 13.40. The value $ka = 10$ corresponds to 1 MHz and transducer radius $a = 0.25$ cm. The value $ka = 20$ corresponds to $f = 2$ MHz and $a = 0.25$ cm or $f = 1$ MHz and $a = 0.5$ cm. Value $ka = 40$ corresponds to 4 MHz and $a = 0.25$ cm or $f = 2$ MHz and $a = 0.5$ cm, the case examined in Fig. 13.14

- Extracting information depends also on the shape of the transducer.
- Shaping the face of transducer may improve the spatial resolution and increase strength of returned echo.
- Medically, the impedance of a typical transducer is 30 MPa s/m, so an impedance matching material is needed to image through patient's skin.

$$I \propto 1/r^2 \left(\frac{J_1(ka\sin\theta)}{ka\sin\theta} \right)^2$$

Pulse Echo Imaging

- A short pulse (typically 0.5 μs with frequency about 5 MHz) applied to tissue by piezoelectric transducer. The pulse speed is about c = 1540 m/s (or 1.54 mm / μs).
- When the acoustic impedance of tissue are different, part of the pulse is reflected as an echo which can be detected by the same outputting piezo transducer. Generally the distance from source to boundary is $\Delta x = \frac{c\Delta t}{2}$.
- The echo intensity versus time is an *A scan*. Adding different directions, and plotting the brightness as the intensity of the echo, versus position in the body in the plane of the scan is a *B scan*.

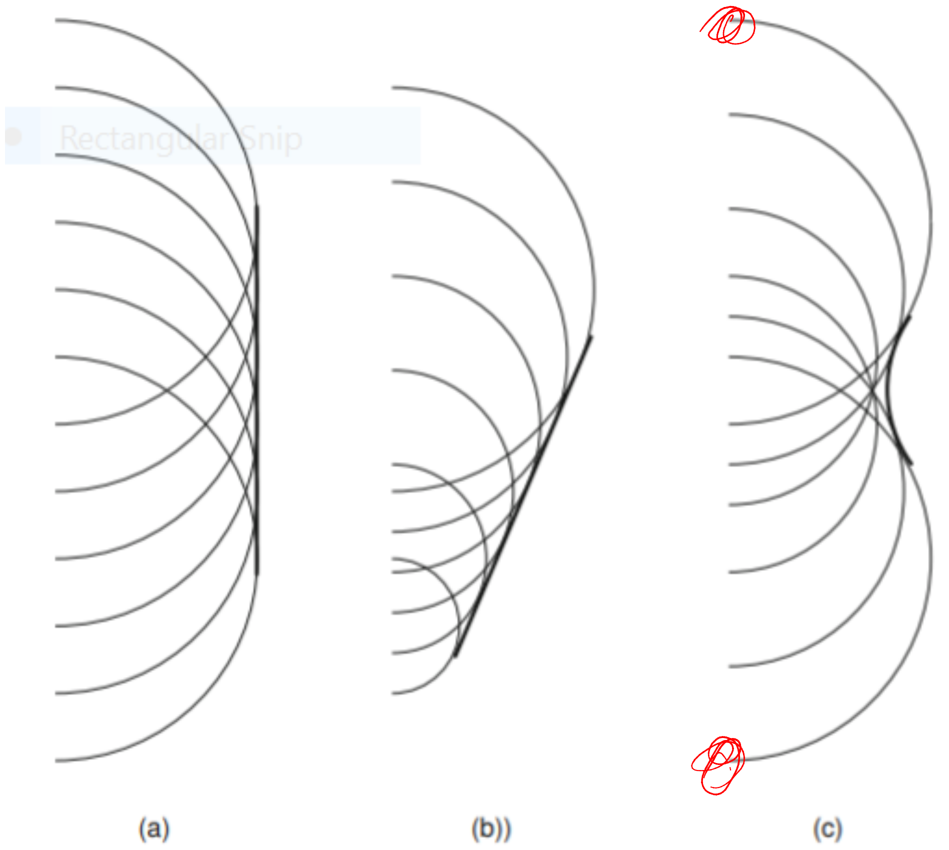
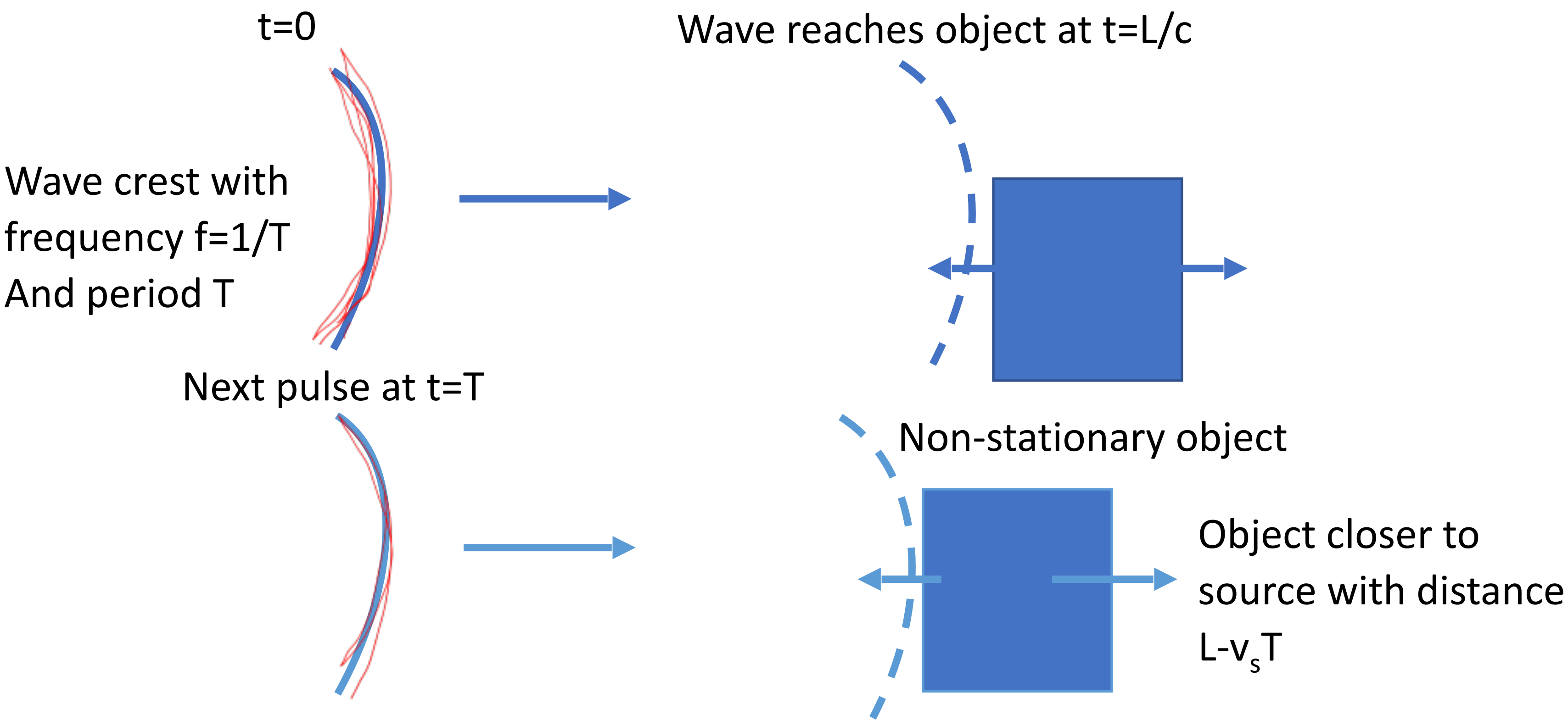


Fig. 13.18 How a phased array or delayed-pulse array works. Five transducers have been pulsed; the semi-circles show the propagating lines of constant phase from each one. The thick lines show the advancing wave front. In (**a**) all five transducers have been pulsed at the same time. The signals from each transducer add along the plane wave front traveling to the right. In (**b**) the top transducer was pulsed first. Each lower transducer was pulsed at successively later times, so the pulses have not traveled as far. This steers the beam downward. In (**c**) the outer transducers were pulsed first. As one goes inward, each transducer was pulsed later than the one before. This focuses the beam. The same technique can be used to steer or focus the sensitivity to the scattered wave during detection

Doppler Effect in Ultrasound

- Doppler Effect describes effects that occur when the source of an ultrasound wave is moving: the frequency of the wave observed by the stationary receiver is different than the frequency of the source.



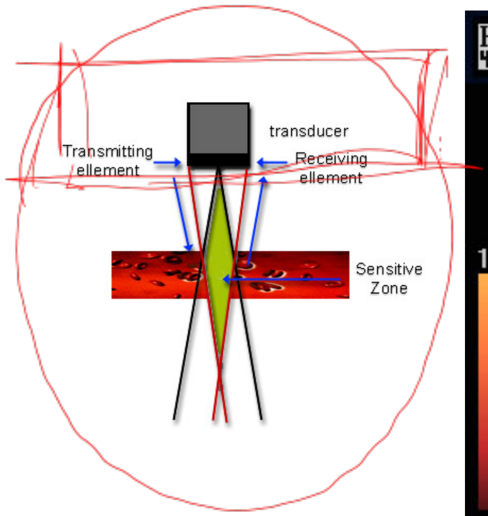
The time to reach receiver is $t=T+(L-v_sT)/c$.

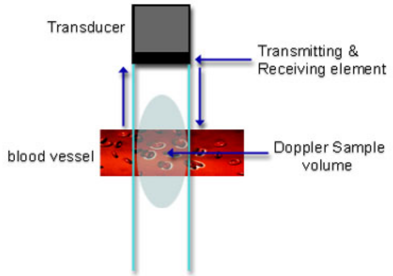
And the time between crests seen at the receiver is $T'=T+(L-v_sT)/c-L/c=T(1-v_sT)$.

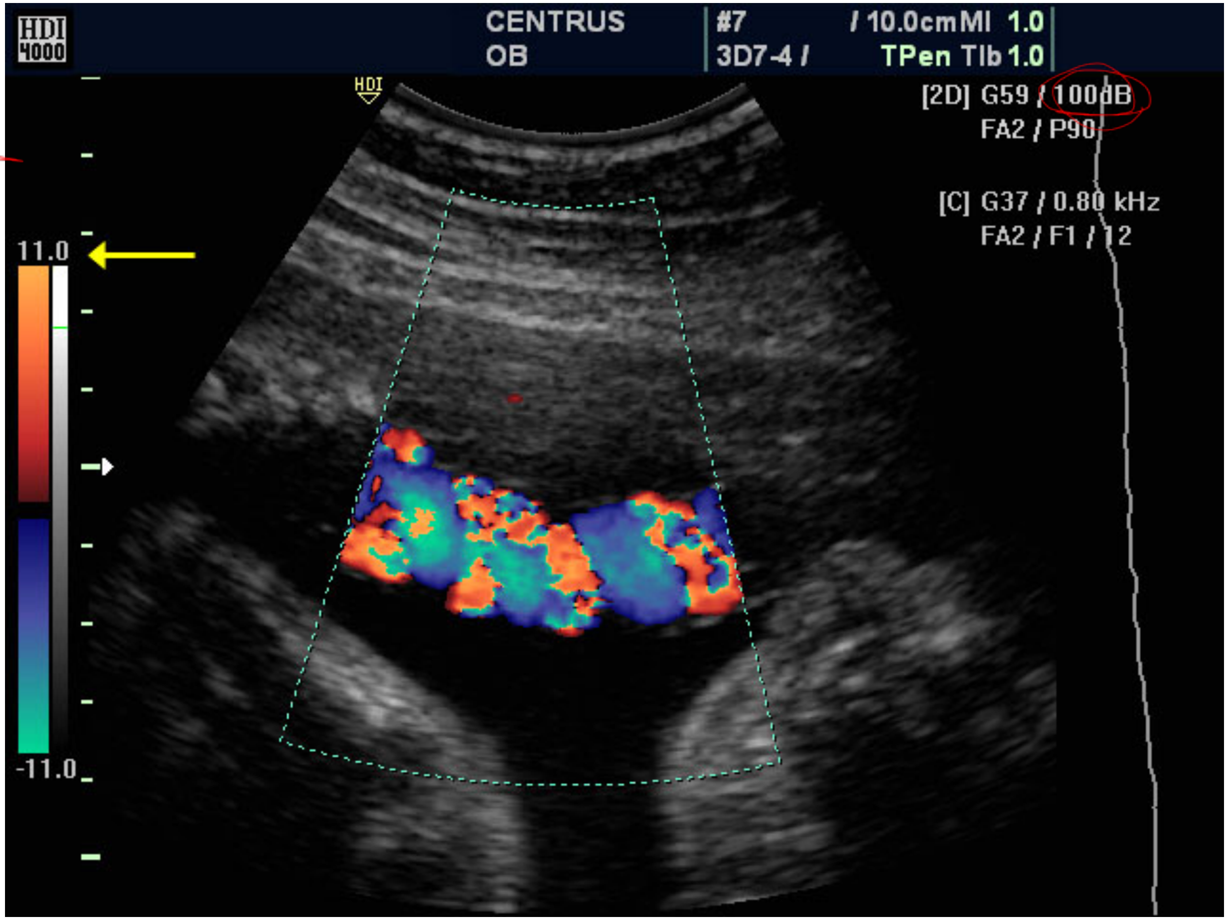
The frequency observed by receiver if object moving towards source $f'=1/T'=f/(1-v_s/c)$

Where f is the frequency produced by source and f' is the frequency received after reflection $f'=f(1-v_s/c/1+v_s/c)$

Doppler Effect in Ultrasound







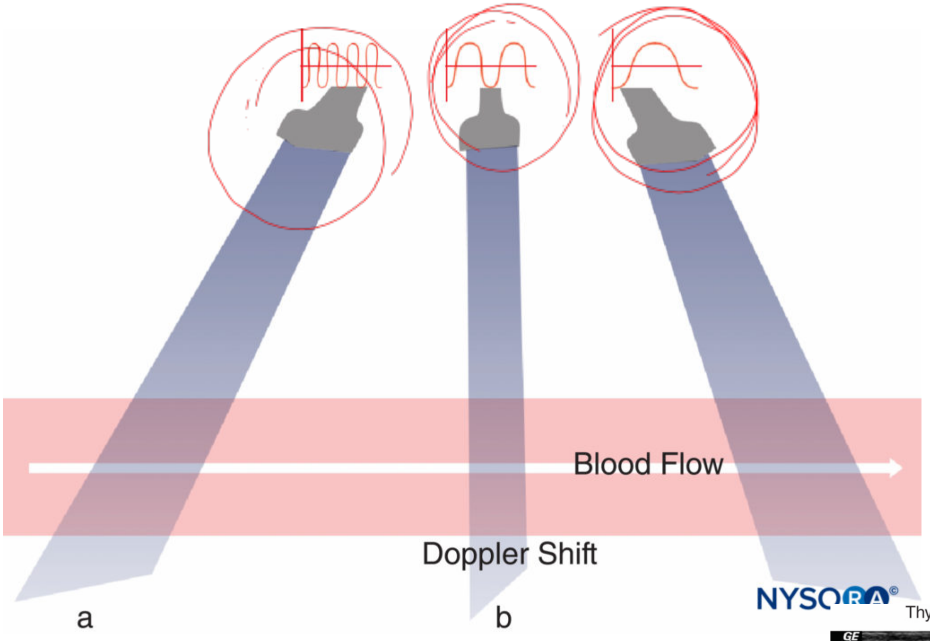
The Doppler effect allows to measure the blood flow in the arteries and vein in the umbilical cord (though ambiguity which is which in the color scale). In different images taken at different angle, with location known, the red corresponds to artery and blue to vein.

4 kHz, power and attenuation to 100 dB, controlled

Noise and depth precision…

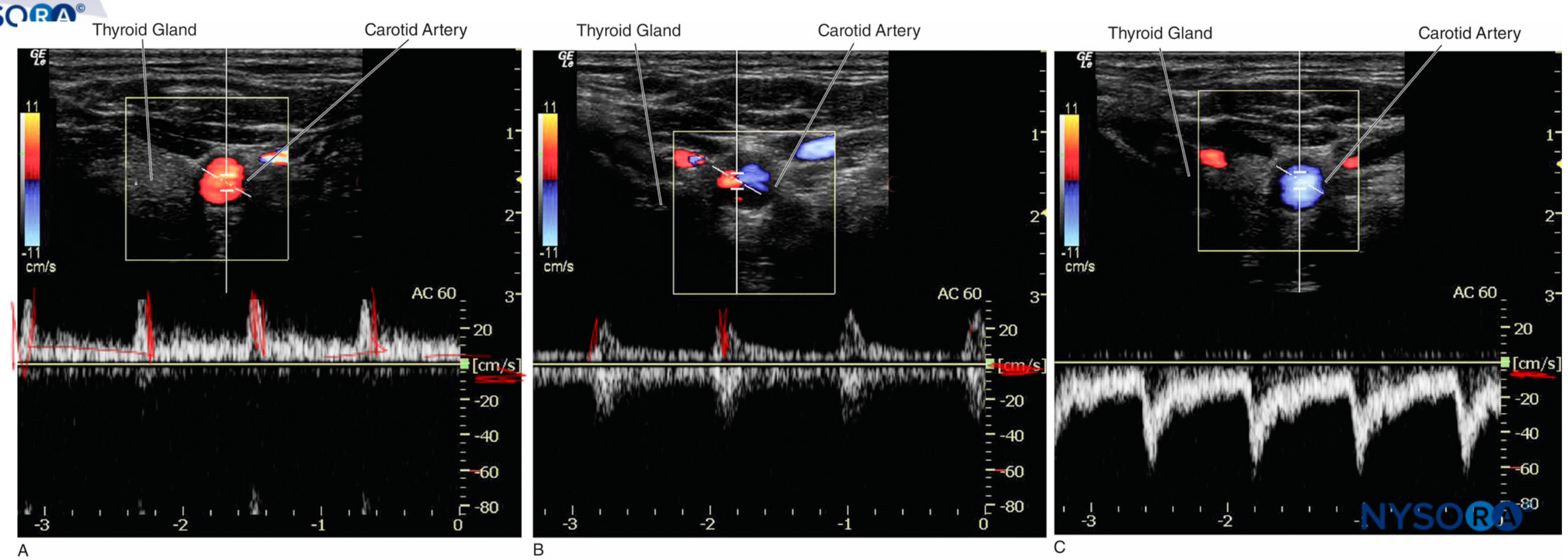
https://sonoworld.com/Client/Fetus/html/doppler/capitulos-html/chapter_01.htm

Doppler Effect in Ultrasound



Color Doppler tells the location of veins or arteries (depending on defining convention)

Power Doppler can identify smaller blood vessels, though at expense of direction and speed information.



Therapeutic Uses (and Dangers)

- Diathermy - Heating from the sound wave energy
- Cavitation – when tiny bubbles of steam form and then collapse from high intensity sound waves
- Lithotripsy – destruction of kidney stones with sharply focused ultrasound