Modern Physics

Topics in Medical Physics: Imaging

X-ray Absorption coefficients of different elements and biological tissue

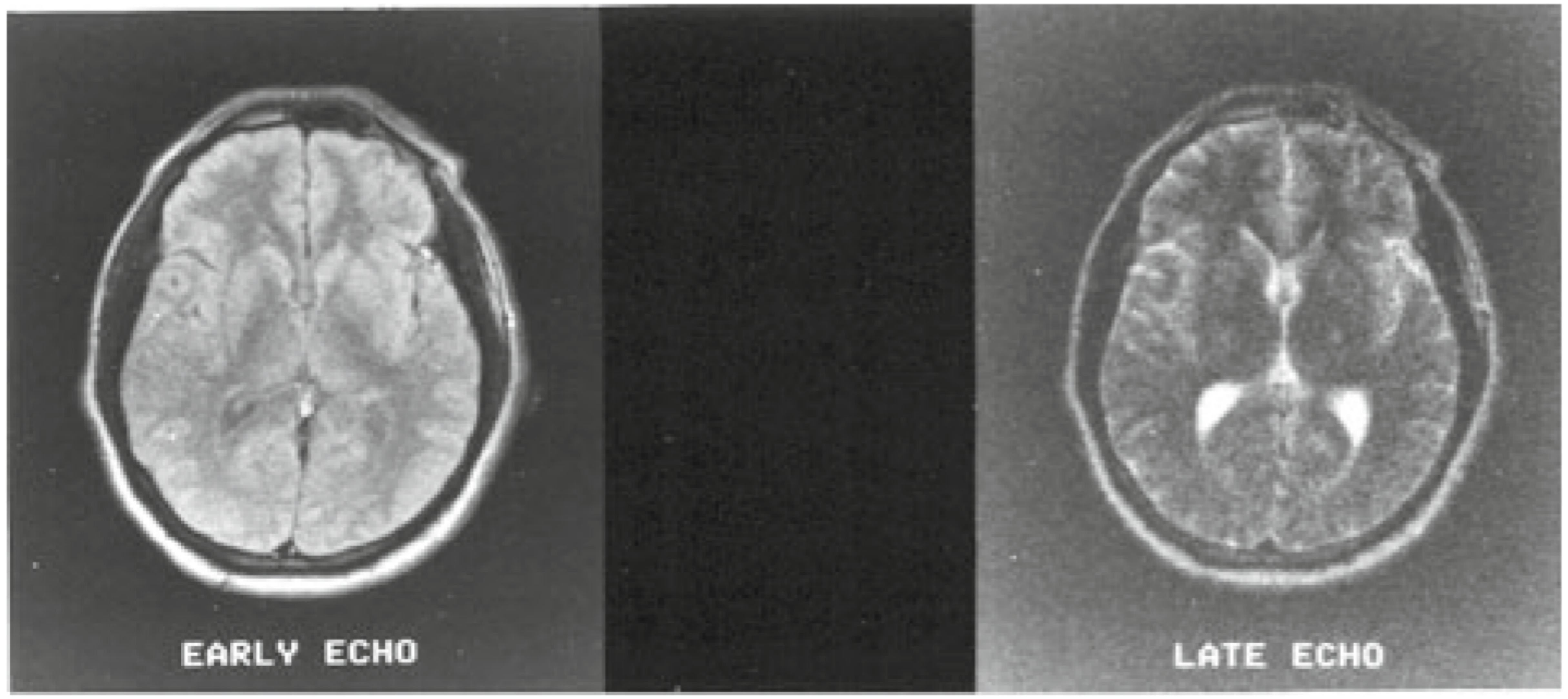
± From considering Beer-Lambert law, $I=I_0e^{-\alpha*x}$

Material	**Atomic #**	**Absorption**	**Specific Absorption**
		Coefficient, α (cm^{-1})	**Coefficient, α/ϱ (cm^2/g)**
Fat	–	0.1788	0.196
Muscle	–	0.2045	0.2045
Brain		0.2061	0.2061
Bone		0.466–0.548	~0.28 cm^2/g since bone ϱ correlates with α
Al	13	131	48.7
P	15	132	73
Ca	20	266	172
Cr	24	1.86×10^3	259
Fe	26	2.55×10^3	324
Co	27	3.19×10^3	354
Pb	82	2.73×10^3	241

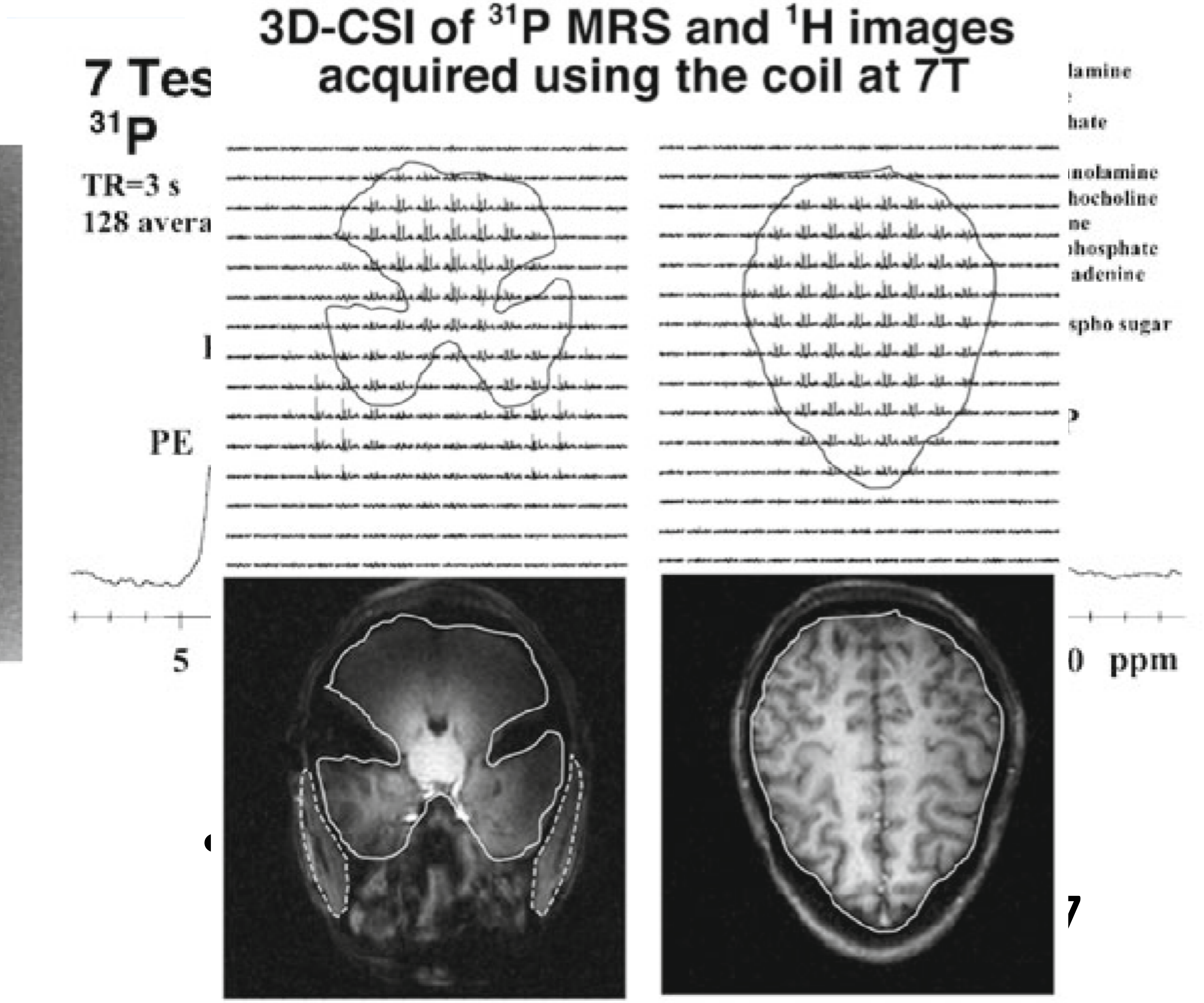
Nuclear Magnetic Resonance – Magnetic Resonance Imaging

- What we’ll cover in this discussion:
  - What does NMR/MRI do medically?
  - Basics of nuclear magnetic resonance
  - New directions currently in literature

Examples of What Can Be Measured in Human Physiology

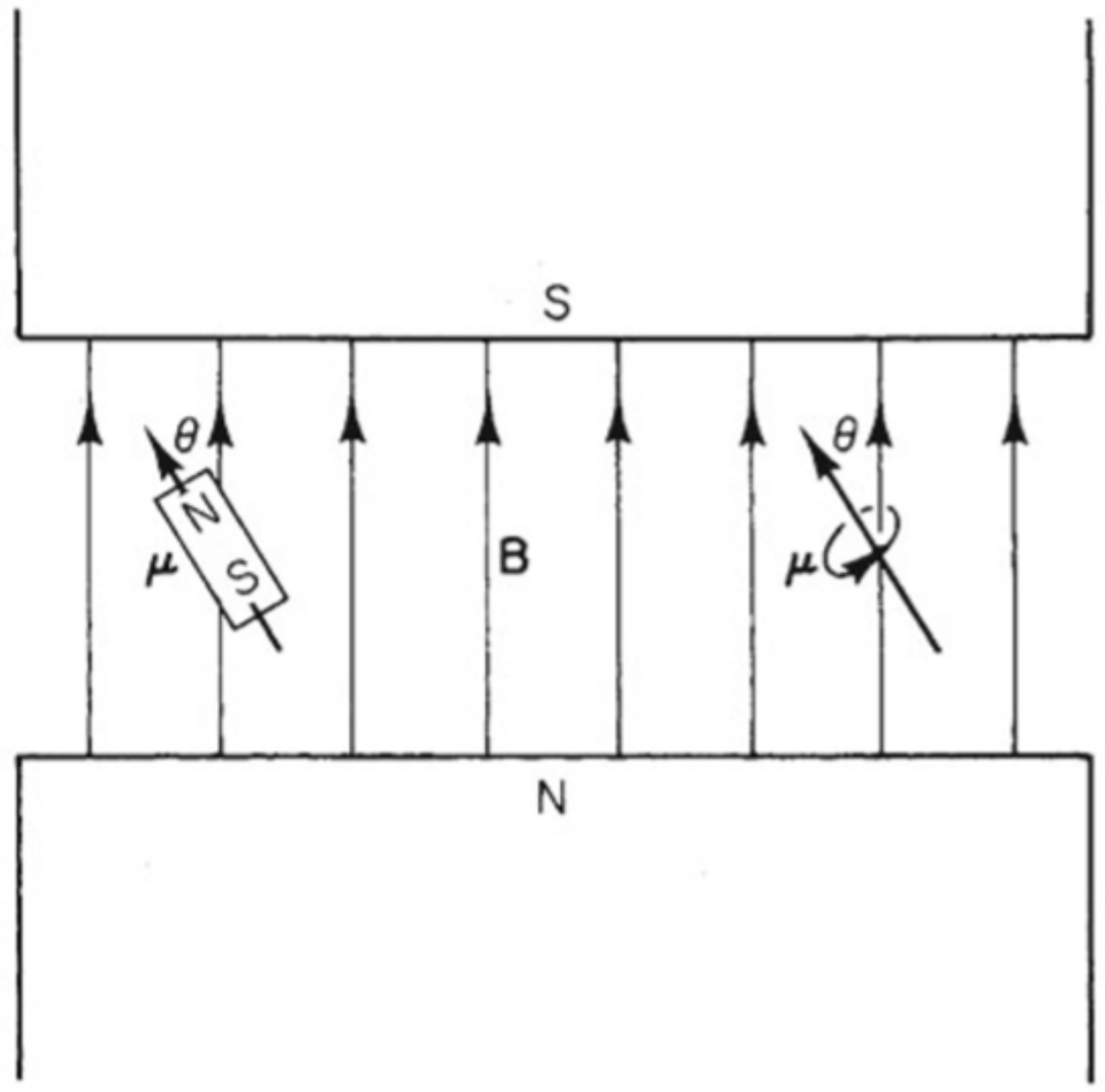


- Image of brain at different echo time, showing contrast in brightness arising from T1 values in the cells at location.



The Source of the Magnetic Moment

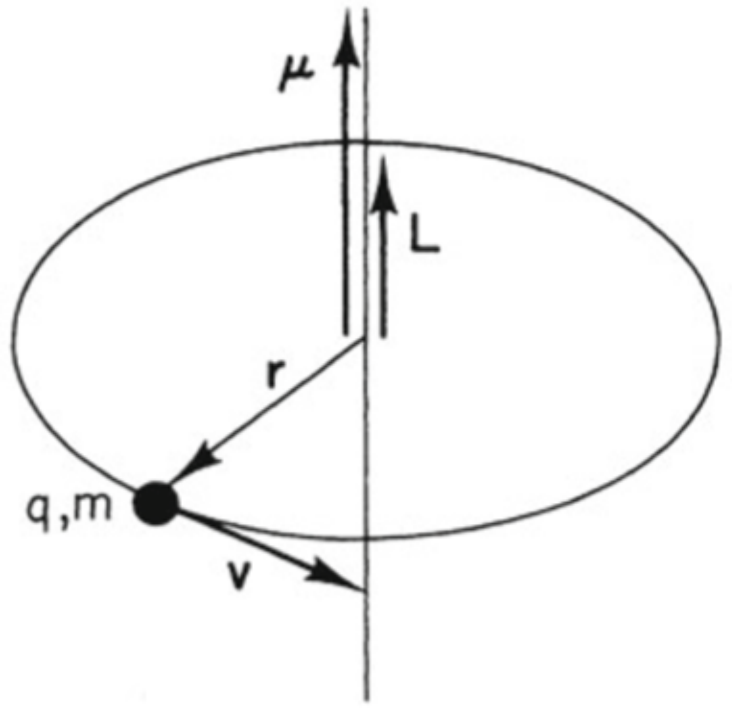
Particle	Spin	$\Gamma=\omega_{Larmor}/B$ (s^{-1} T^{-1})	ν/B (MHz T^{-1})
Electron	½	1.7608e11	2.8025e4
Proton	½	2.6753e8	42.5781
Neutron	½	1.8326e8	29.1667
23Na	½	0.7076e8	11.2618
31P	½	1.0829e8	17.2349



$$\tau = \mu \times B$$

$$\mu = \gamma \times L$$

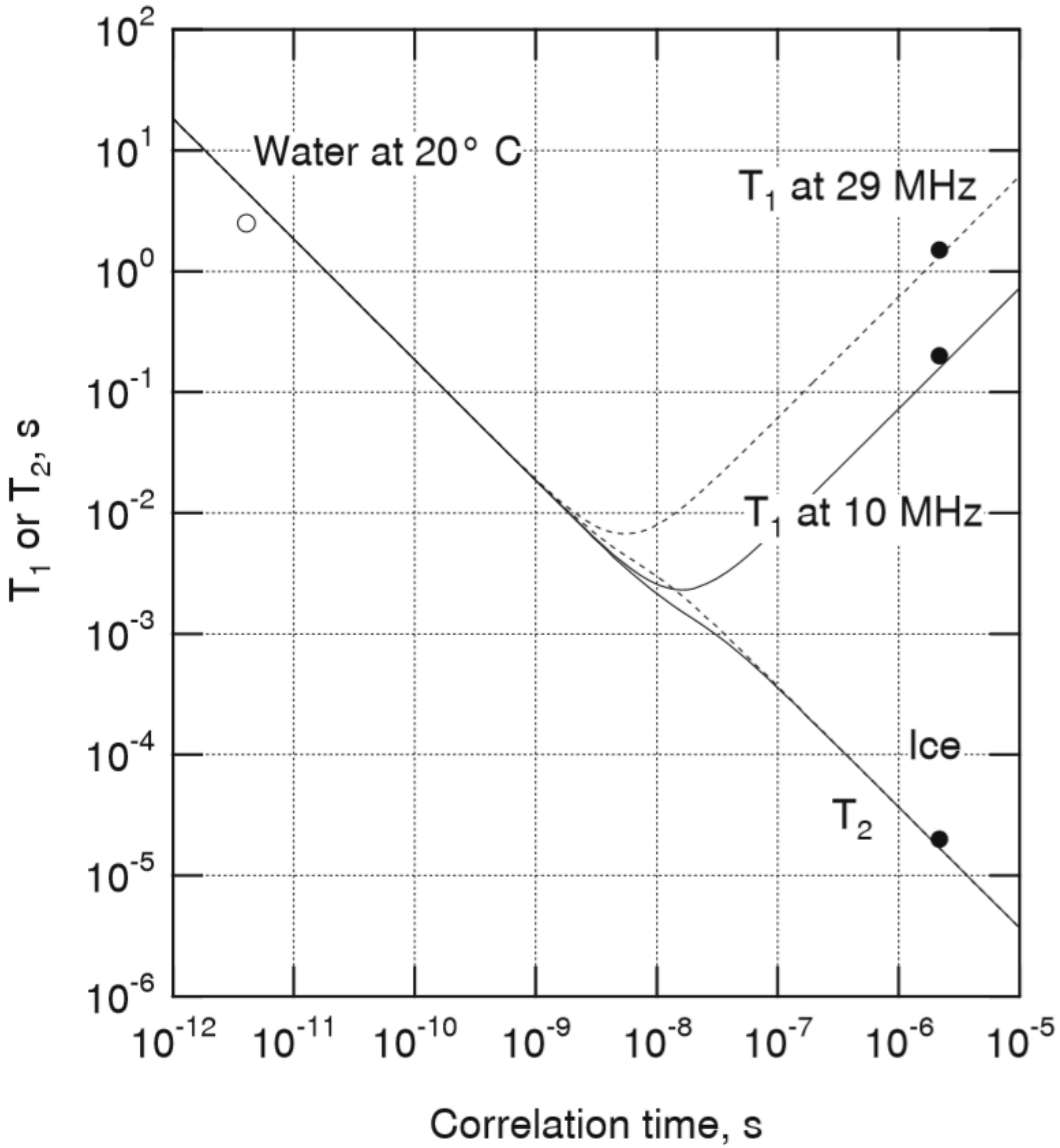
$$\gamma(L \times B) = \frac{dL}{dt} \ or \ \gamma\left(\mu \times B\right) = \frac{d\mu}{dt}$$

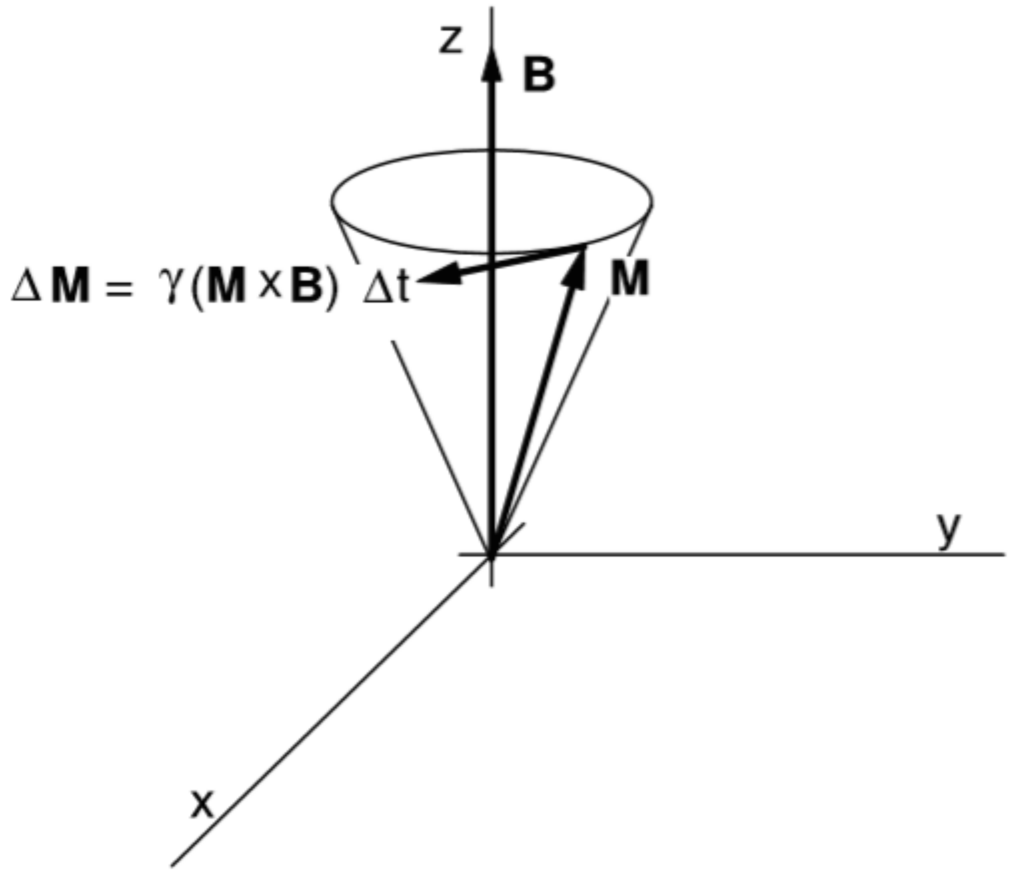


Approximate Relaxation Times

At 20 MHz	T1 (ms)	T2 (ms)
Whole Blood	900	200
Muscle	500	35
Fat	200	60
Water	3000	3000

Adding 20 ppm Fe3+ to water reduces T1 from 3000 ms to 20 ms.





rewriting

$$\frac{dM}{dt} = \gamma(M \times B)$$

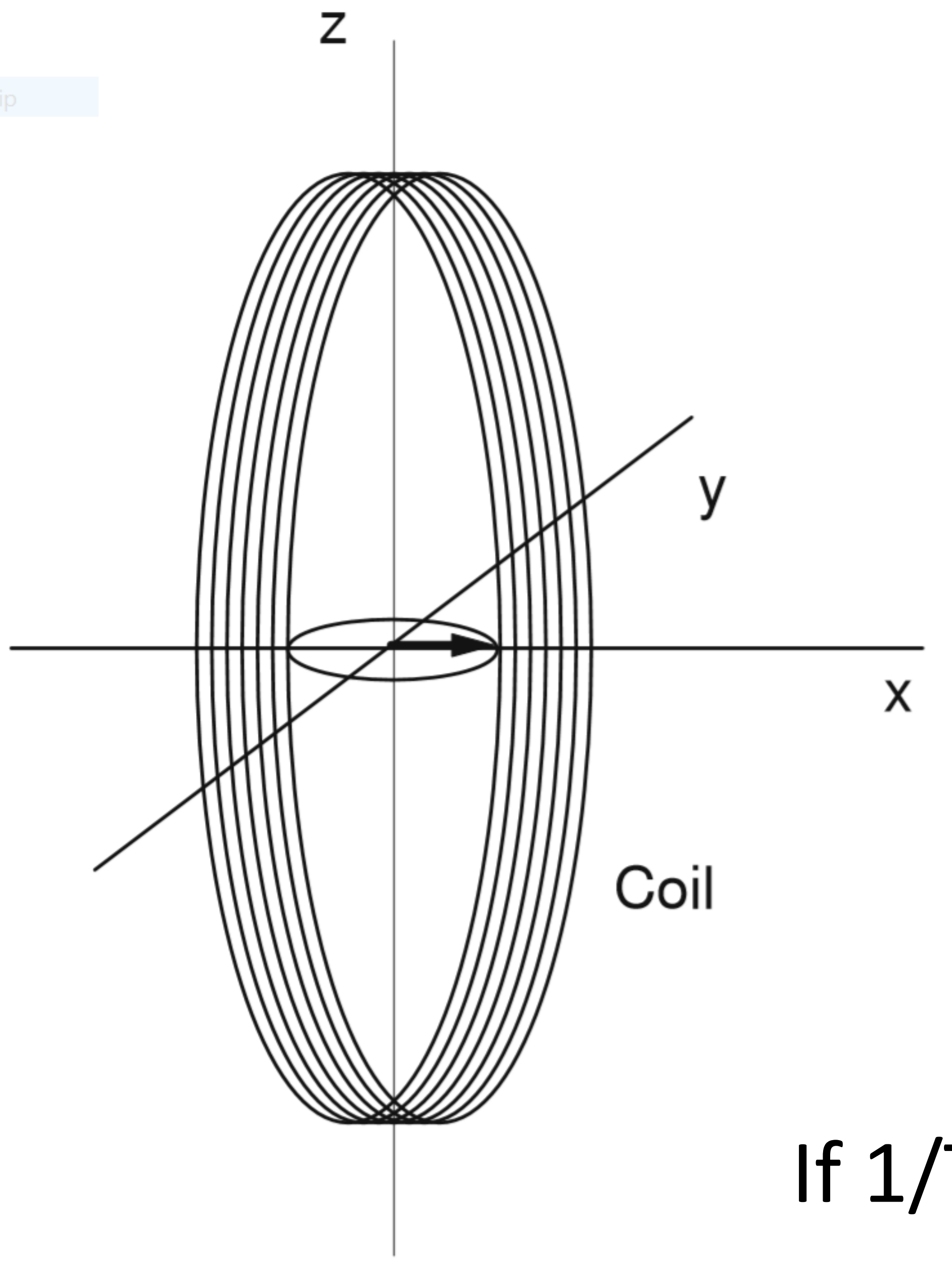
$$\frac{dM_z}{dt} = \frac{1}{T_1}(M_0 - M_z)$$

Longitudinal relaxation

$$\frac{dM_{x,y}}{dt} = -\frac{M_{x,y}}{2}$$

Detecting the Magnetic Resonance Signal

- Nuclear spins in a strong magnetic field have induced magnetic moments.
- Varying magnetic field causes the moment to precess about the static magnetic field (nutate is like massaging).



$$\text{voltage} = -\frac{\partial \Phi}{\partial t} = -\frac{\partial}{\partial}\iint \boldsymbol{B} \bullet d\boldsymbol{S}$$

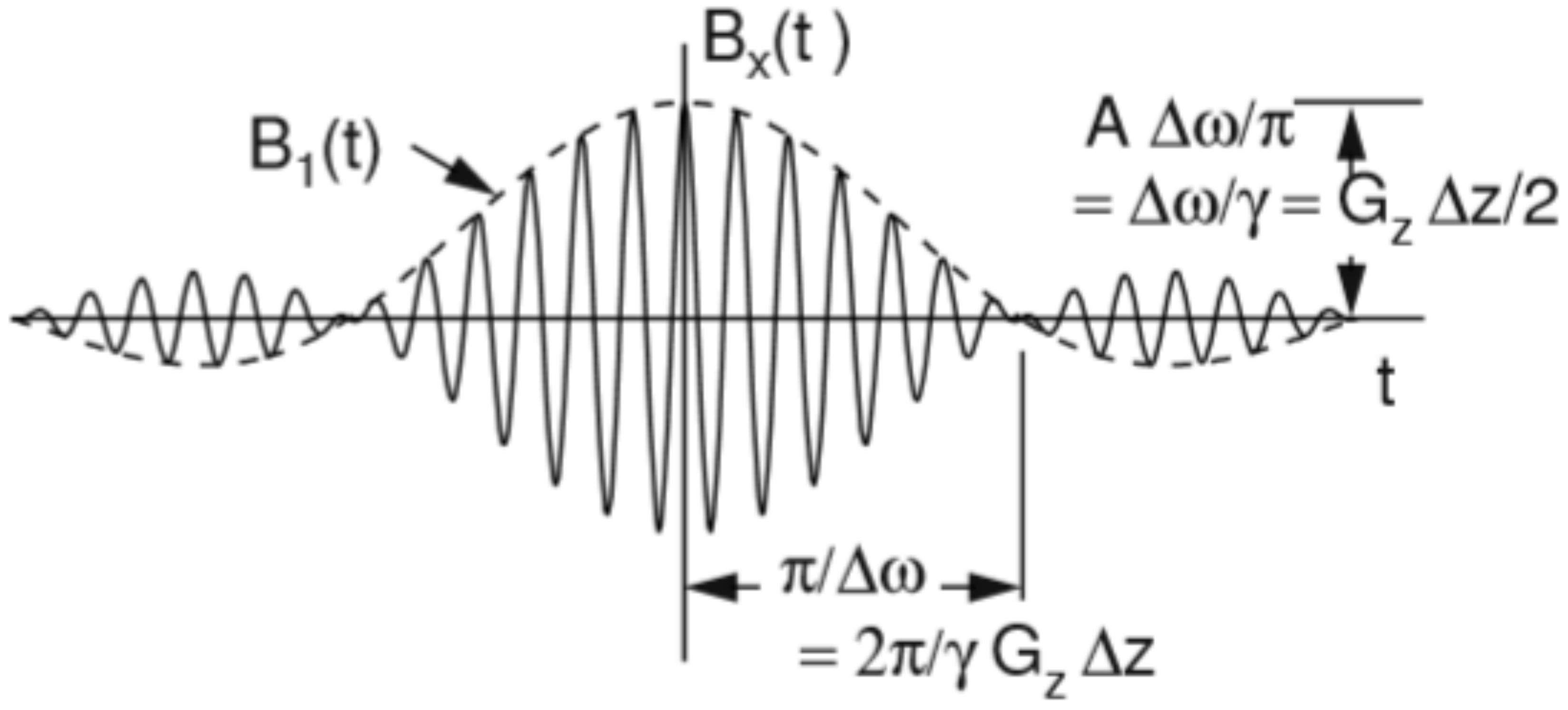
$$\Phi = -\int B_r 2\pi a^2 sin\theta d\theta = -\frac{\mu_0}{4\pi}\frac{4\pi\mu_x}{a}\int_0^{\frac{\pi}{2}} cos\theta sin\theta d\theta = -\frac{\mu_0}{4\pi}\frac{2\pi\mu_x}{a}$$

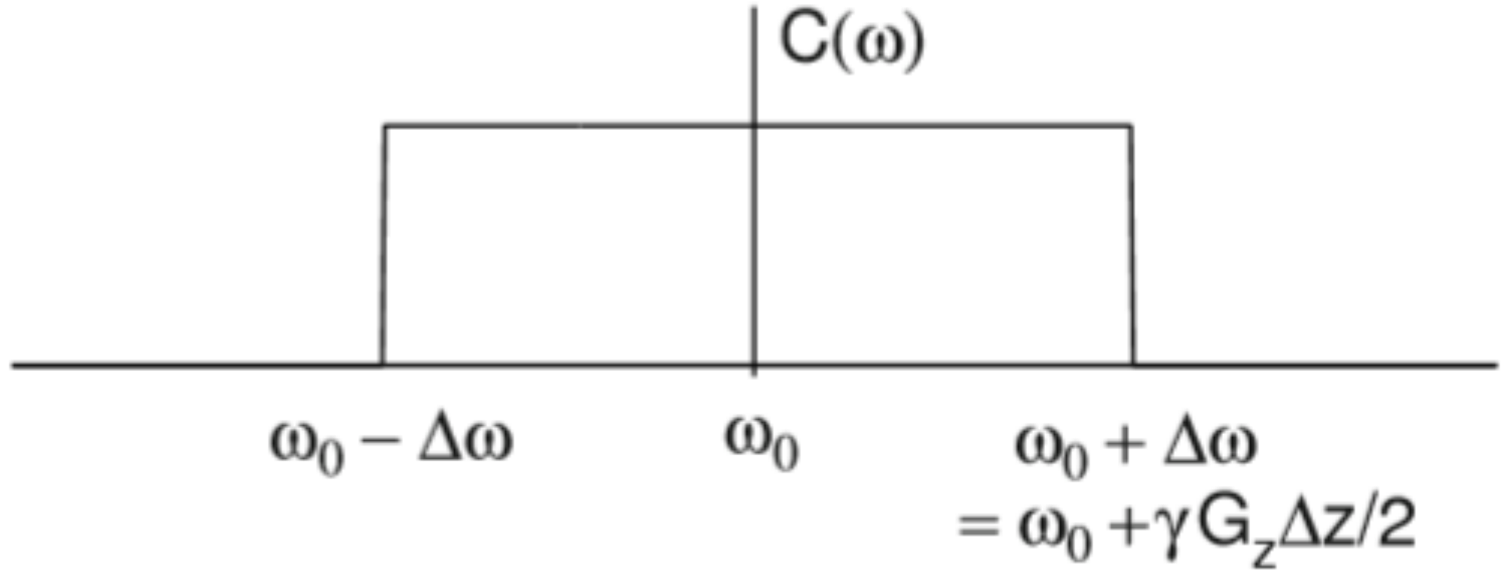
For the flux for a magnetic moment, $m = M\Delta V$, the induced voltage is then

$$voltage = \frac{\mu_0}{4\pi}\frac{2\pi M_0 \Delta V}{a} e^{-\frac{t}{T_2}}\left(-\frac{1}{T_2}\cos\left(-\omega_0 t\right) + \omega_0 \sin(-\omega_0 t)\right)$$

If $1/T_2 << \omega_0$, and for spin ½ particles, $$voltage = \frac{\mu_0}{4\pi}\frac{\pi N \Delta V \gamma^3 \hbar^2 B_0^2}{2 k_B T\, a} e^{-\frac{t}{T_2}}\left(\sin(-\omega_0 t)\right)$$

Max value of B1, the envelope





In a typical machine, Gz =5×10−3 Tm−1. Assuming a slice thickness Δz = 0.01 m, the Larmor frequency across the slice varies from ω0−Δω to ω0+ Δ ω, where Δω = γGz Δ z/2= 6.68×103 s−1 (Δ f =1.064 kHz).

$$B_{x(t)} = \frac{A}{2\pi} \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} \cos(\omega t)\, d\omega = \frac{A\Delta\omega}{\pi} \frac{\sin(\Delta\omega t)}{\Delta\omega t} \cos(\omega_0 t)$$

The angle of nutation is

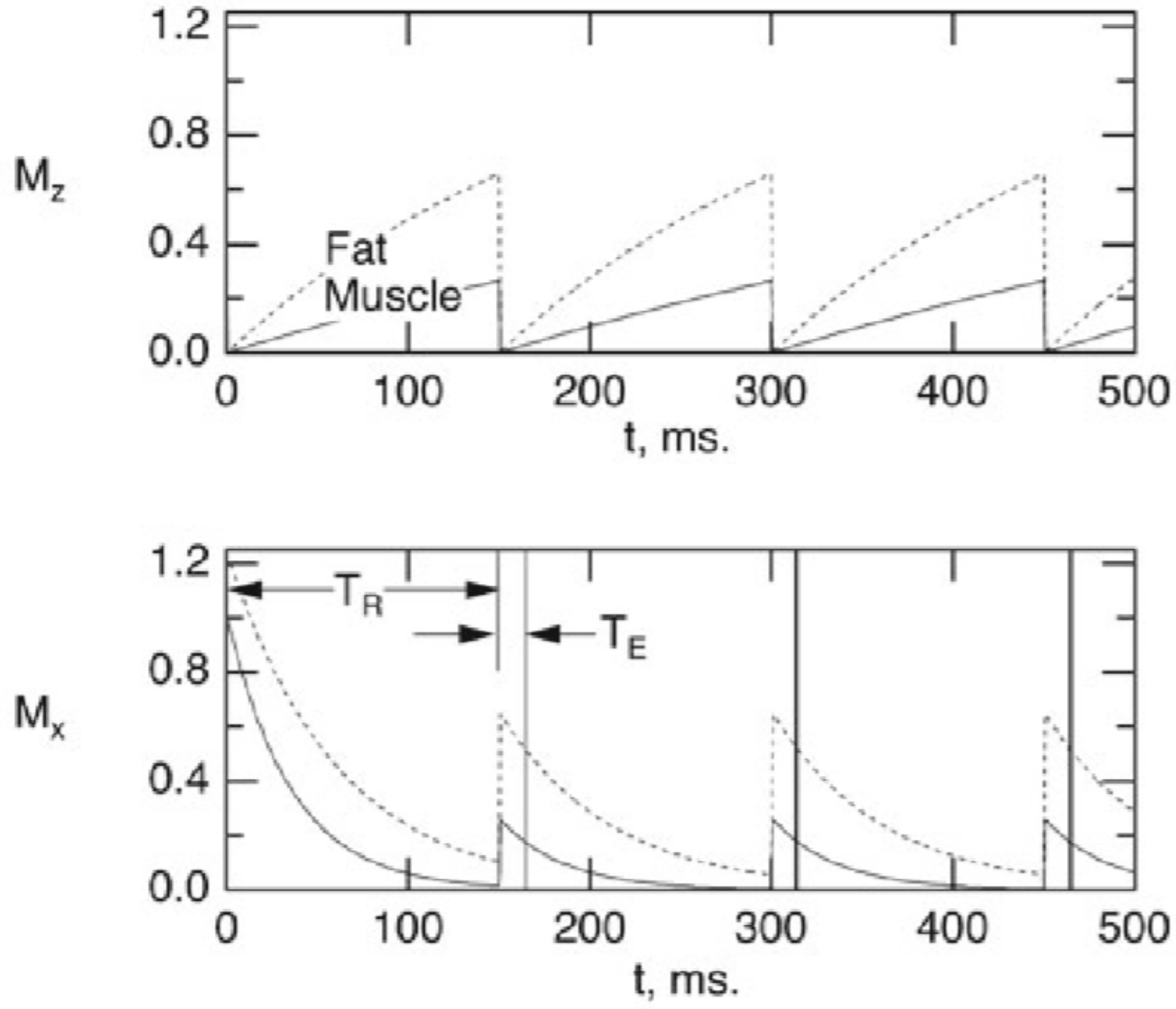
$$\phi = \int_{-\infty}^{\infty} \omega_1(t)\, dt = \frac{\gamma}{2} \int_{-\infty}^{\infty} B_1(t)\, dt = \frac{\gamma A\ \Delta\omega}{2\ \pi} \int_{-\infty}^{\infty} \frac{\sin(\Delta\omega t)}{\Delta\omega\ t} dt = \frac{\gamma A}{2}$$

Image Contrast and Pulse Parameters

- Magnetic moment in the sample at the time of measurement, taking into consideration longitudinal and transverse relaxation:

$$M(T_R, T_E) = M_0\left(1 - 2e^{\frac{T_R}{T_1} + \frac{T_E}{2T_1}} + e^{-\frac{T_R}{T_1}}\right)e^{-\frac{T_E}{T_2}}$$

- Muscle typically M_0 = 1.02 (arb. Units) and T_1 = 500 ms, T_2 = 35 ms
- Fat typically M_0 = 1.24, T_1 = 200 ms, T_2 = 60 ms.



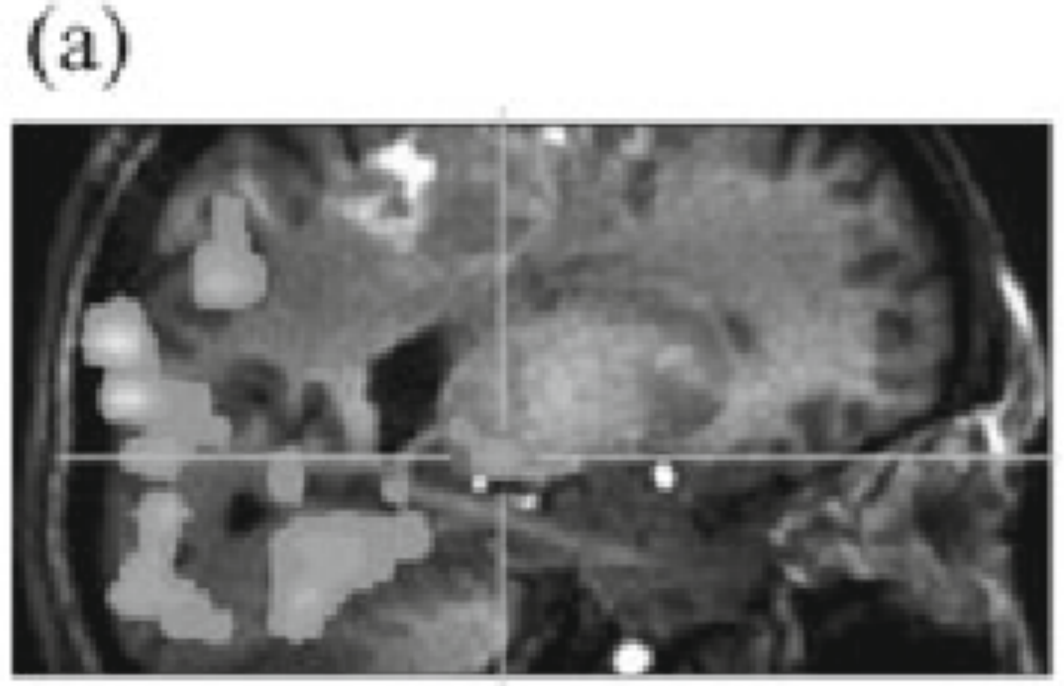
For short T_E, the image density depends primarily on T_1.

Estimates of Helium Consumption

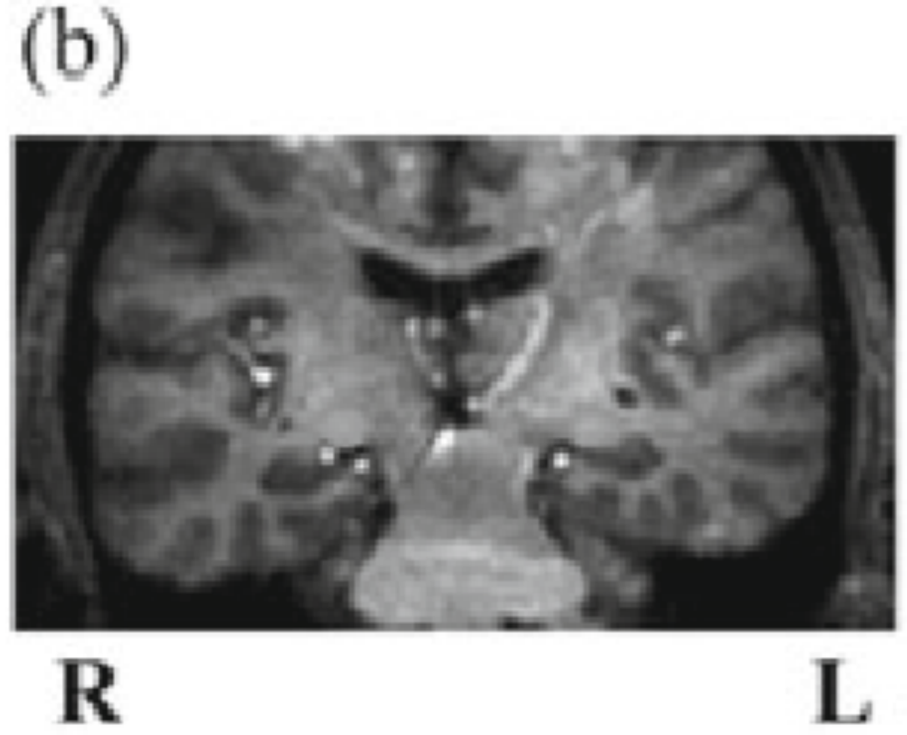
- Varies machine to machine, assume for example around 1800 to 2000 L consumption per year for one machine (with 0.5 T to up to 3 T or so capability).
- This amounts to around USD15 per liter * 1800 L = USD27000 to run an MRI machine. (upper bound estimates, since refill usually occurs at for example 50-65% helium level usage, fluctuations in helium price from half to double)

Functional MRI

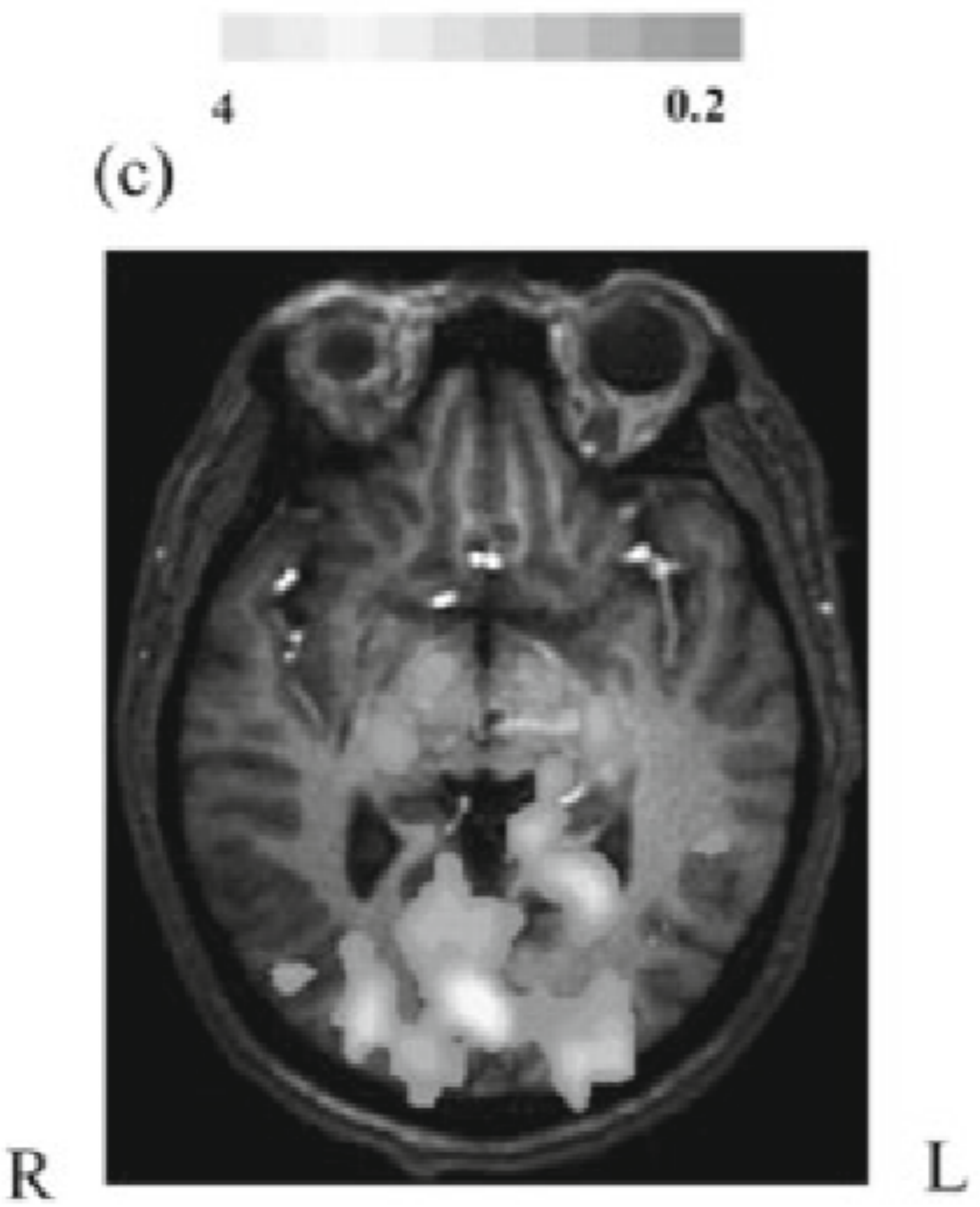
- Oxygenated hemoglobin is less paramagnetic than deoxyhemoglobin.
- No other external contrast required.
- Other contrast agents such as gadolinium shielded by a complex molecule, used to measure pH and ions such as Zn, Ca, Cu, and enzymes. ^{19}F is also being tested as alternative to Gd.
- In brain – one research direction is utilizing biomagnetic response of brain activity, another is utilizing Lorentz force to move currents to regions with different Larmor frequency.



Sagittal or side view



Coronal or front view



axial view, from below

Bright spots show activity in visual cortex and areas between cortex and eyes, 4 T magnet.

Beware the choices of elements to use in human physiology!