Modern Physics – Topics in Medical Physics

Topics Covered

- Mechanics
- Buoyancy
- Compressibility
- Viscosity
- Human Circulatory System
- Systems of Large Particles
- Scaling Laws
- Gibbs Free Energy
- Chemical Potential



Fig. 1.10 Forces on the foot, neglecting its own weight

• All the forces have to be balanced.

From Hobbie and Roth



F: net force of the abductor muscles, acting on the greater trochanter. Muscles primarily the gluteus medius and gluteus minimus.
R: force of the acetabulum (the socket of the pelvis) on the head of the femur.
N: upward force of floor on the bottom of foot (equal to weight).
W_L: weight of leg, acting vertically downward at

the center of gravity of the leg. Some sources indicate $W_L \sim W/7$.

$$\sum_{w_{7}} F_{y} = Fsin(70^{\circ}) - R_{y} - \frac{W}{7} + W = 0$$

$$\sum_{r_{x}} F_{x} = Fcos(70^{\circ}) - R_{x} = 0$$

$$\sum_{r_{x}} \tau = -Fsin(70^{\circ})(7) - \left(\frac{W}{7}\right)(10 - 7) + W(18 - 7) = 0$$

$$F = 1.6W \text{ and } R_{x} = 0.55W \text{ and } R_{y} = 2.36W$$

$$tan\phi = \frac{R_{x}}{R_{y}} = 0.23 \text{ or } \phi = 13^{\circ}$$



ponds, lakes, rivers, streams, aquifers 0-.5 ppt

estuaries, mangrove swamps, brackish seas and lake, brackish swamps .5-30 ppt

saline water seawater, salt lakes 30-50 ppt

brine pools

50+ ppt

water

Buoyancy $F = \left(-\frac{dp}{dz} - \rho g\right) dx \, dy \, dz$

- Buoyancy is the tendency for objects in "fluids" to resist the fluid itself, and depends on the density of objects in play.
- The total buoyant force experienced by an object with density ρ can be described by $F = \left(\rho_{fluid} \rho\right)gV$
- Mammals are typically made of mostly water and some air.
- For humans, average densities are 500 950 kg/m³. Considering surface seawater densities, we can see why human bodies typically float in the ocean (positive force pushes upwards in z direction).





- Salinity of the ocean water (image from <u>Welcome1To1The1Jungle</u> on Wikipedia), as measured in practical salinity units
- At the surface, seawater ranges from 1020 to 1029 kg/m³, dependent on temperature and salinity and geographical position



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Seawater elemental composition at salinity 3.5% on Earth

Element	Percent by mass
Oxygen	85.84
Hydrogen	10.82
Chlorine	1.94
Sodium	1.08
Magnesium	0.1292
Sulfur	0.091
Calcium	0.04
Potassium	0.04
Bromine	0.0067
Carbon	0.0028
Vanadium	1.5*10 ⁻¹¹ to 3.3*10 ⁻¹¹

Compressibility

• Is defined as a change in the volume of an entity from external pressure

 $\frac{\Delta V}{V} = -\kappa \Delta p$

- The dimensionless number κ, has units N⁻¹m² or Pa⁻¹ and is the inverse of the bulk modulus.
- Other units used are the microsip, defined as 1*10⁻⁶ psi⁻¹.

Liquid	Compressibility				
	Pa-1 x 10-11	Atm-1 x 10 ⁻⁶			
Carbon disulfide	93	94			
Ethyl alcohol	110	111			
Glycerine	21	21			
Mercury	3.7	3.8			
Water	45.8	46.4			
Loose sand	10,000 to 5200 (1*10 ⁻⁷ to 5.2*10 ⁻⁸)				
Dense sand	2000 to 1300 (2*10 ⁻⁸ to 1.3*10 ⁻⁸)				

Some data from Sears, Zemansky, Young, and Freedman, University Physics, 10th Ed., Section 11-6.

Compressibility of Oils Versus Gases

For example, let's consider the compressibility of some oils under isothermal conditions: rewrite the equation above as.

The oil expands in a cell volume by reducing pressure (?!) and concomitant oil volume increase and oil pressure decrease are monitored. At the bubble-point pressure, free gas accumulates in the cell.



 $c_0 = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_{T}$

Viscosity

• Viscosity of a material arises from sheared stress, a force that is in directions perpendicular to the original movement.

$$\frac{F}{S} = \eta \frac{dv_x}{dy} = \eta \frac{d\epsilon_s}{dt}$$

• The second term describes the force gradient as a function of the rate of change of the shear strain. $v_x = v$



Water

• That ever most valuable resource, water, is a Newtonian fluid.



Measurement from Anton Paar, <u>https://wiki.anton-paar.com/en/water/</u>



Common Viscosities of Glass and Materials



Substance	Viscosity (mPa s)	Tempreature (Celsius)
Benzene	0.604	25
Water	1.0016	20
Mercury	1.526	25
Whole Milk	2.12	20
Olive Oil	56.2	26
Honey	2000-10000	20
Ketchup	5000-20000	25
Peanut Butter	1*10 ⁴ to 1*10 ⁶	
Pitch	2.3*1011	10-30

From wikipedia

Human Circulatory System

 Sphygmomanometer measures blood pressure, systolic/ diastolic (maximum pressure during cardiac cycle/ minimum pressure during cardiac cycle). A typical reading gives 110/70 torr.

•
$$R = \frac{\Delta p}{i}$$
 in torr ml⁻¹ min or Pa

- $=\frac{8\eta\Delta x}{\pi R_p^4}$ (*if Poiseuille*) m⁻³ s (SI). The vascular resistance (as measured globally) is the pressure difference in the "pipes" measured from the contributing current (blood) flow in that pipe region.
- The mathematical manipulation of vascular resistance is similar to electrical resistance:

•
$$R_{tot} = R_1 + R_2 + R_3...$$
For vessels in series
• $\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}...$ For vessels in parallel



Velocity Profile in a Pipe

 Consider laminar viscous flow of fluid through a pipe of constant radius Rp. Force balance gives



In units of m³s⁻¹. Note that r*dp/dx is the force to area ratio, with viscosity in units N s m⁻² or kg m⁻¹s⁻¹ or Pa s. An older unit is the dyn cm⁻² or poise (1 poise = 0.1 Pa s).

Human Circulatory System

 As a modle, consider an incompressible fluid during time Δt with velocity v, the volume fluence is (volume transported) SvΔt

$$j_v = \frac{(volume\ transported\,)}{(area)(time)} = \frac{Sv\Delta t}{S\Delta t} = v$$



Flow of fluid across the plane at B

• The volume current is i, integrated over the cross sectional area of the pipe, for Poiseuille flow.

$$i = \int_{0}^{R_{p}} j_{v}(r) 2\pi r \, dr = \frac{2\pi}{4\eta} \frac{\Delta p}{\Delta x} \int_{0}^{R_{p}} \left(R_{p}^{2} - r^{2} \right) r dr = \frac{\pi R_{p}^{4}}{8 \eta} \frac{\Delta p}{\Delta x}$$

Note the compressibility is defined as $\frac{\Delta V}{V} = -\kappa \Delta p$ with units Pa⁻¹ or N⁻¹ m². Integration Compressibility of water is 5x10⁻¹⁰ Pa⁻¹.

Integration by parts $u \int dv = uv - v \int du$

Human Circulatory System: Examples

- As an example, consider a pore with certain size that might be found in the basement membrane of the glomerulus of the kidney.
- Rp = 5 nm, Δp = 15.4 torr = $\rho g \Delta z$ = 2.04*10³ Pa (~15 torr, Hg density 13.55*10³ kgm⁻³), η = 1.4*10⁻³ kg m⁻¹ s⁻¹, Δx = 50 nm. This gives i = 7.2x10⁻²¹ m³s⁻¹.



$$i = \frac{\pi R_p^4}{8 \eta} \frac{\Delta p}{\Delta \mathbf{x}}$$

Is this tiny i what is felt what a stone is passed?! Velocity ~2*10^-11 to even smaller?!

Human Circulatory System



Human Circulatory System

Table 1.4 Typical values for the average pressure at the entrance to each generation of the major branches of the cardiovascular tree, the average blood volume in certain branches, and typical dimensions of the vessels

Location	Average pressure (torr)	Blood volume ^a (ml)	Diameter ^b (mm)	Length ^b (mm)	Wall thickness ^b (mm)	Avg. velocity ^b (m s ⁻¹)	Reynolds number at maximum flow ^c
Systemic circulatio	n						
Left atrium	5						
Left ventricle	100						
Aorta	100	156	20	500	2	0.48	9 400
Arteries	95	608	4	500	1	0.45	1 300
Arterioles	86	94	0.05	10	0.2	0.05	
Capillaries	30	260	0.008	1	0.001	0.001	
Venules	10	470	0.02	2	0.002	0.002	
Veins	4	2682	5	25	0.5	0.01	
Vena cava	3	125	30	500	1.5	0.38	3 000
Right atrium	3						
Pulmonary circulat	ion						
Right atrium	3						
Right ventricle	25						
Pulmonary artery	25	52					
Arteries	20	91					7 800
Arterioles	15	6					
Capillaries	10	104					
Veins	5	215					2 200
Left atrium	5						
^a From Plonsey (19	95)						

From Mazumdar (1992)

Kidney pore

15 torr

^cFrom Milnor (1989)

Bernoulli Equation

• Valid for nonuniform viscous flow that may be laminar or turbulent.

$$p_{1} - p_{2} = \Delta p = \rho \int_{s_{1}}^{s_{2}} \frac{\partial v}{\partial t} ds + \int_{s_{1}}^{s_{2}} -\frac{dp_{visc}}{ds} ds + \int_{s_{1}}^{s_{2}} -\rho v \left(\frac{dv}{ds}\right) + \int_{s_{1}}^{s_{2}} \frac{dp_{grav}}{dz} dz = \rho \int_{s_{1}}^{s_{2}} \frac{\partial v}{\partial t} ds + \int_{s_{1}}^{s_{2}} -\frac{dp_{visc}}{ds} ds + \frac{\rho v_{2}^{2}}{2} - \frac{\rho v_{1}^{2}}{2} + \rho g (z_{2} - z_{1})$$



A flow of air through a Venturi meter, showing the columns connected in a manometer and partially filled with water. The meter is "read" as a differential pressure head in cm or inches of water.

https://en.wikipedia.org/wiki/File:Venturi_Tube_en.webm

https://en.wikipedia.org/wiki/Bernoulli%27s_principle

Scaling Laws Applied to Organisms



Taking H ~ M×.

 Children's (up to > 6 yrs) growth via consumption versus body mass. This model assumes that the mass is proportional to height.



Fig. 2.20 Plot of resting metabolic rate vs. body mass for many different organisms. (Graph is from R. H. Peters 1983. Modified from A. M. Hemmingsen 1960). Used with permission

Plot for different unicellular and sea based organisms at 20 C And homeotherms with body temperature at 39 C.

Energy loss from breathing

- Assuming each breath intakes 0.5 L of air.
- Q = c ρ V (T_{body} T_{surround}) = 1000 J/K*kg * 1.3 kg/m³ * 0.5 L * (37 C 20 C) = 11 J with average power lost to air of 2.2 W (assuming breath every 5 s). For comparison, a typical basal metabolic rate is 100 W.
 - $\rho_{water} = 1000 \text{ kg/m}^3$, $\rho_{air} = 1.3 \text{ kg/m}^3$, $c_{air} = 1000 \text{ J/K}^*\text{kg}$, $c_{water} = 4200 \text{ J/K}^*\text{kg}$. Note assuming no metabolism.
- Note heat capacity is $C = Q/\Delta T$ with units J/K. Specific heat capacity is J/K*kg or J/K*mol.

Large System of Particles

- Considering the forces on all particles in a large system is a very complex problem (although this helps to push efforts in computing science)
- Instead of solving the force on each molecule or particle, traditionally consider the state of the entire system and the statistics of these ensembles of particles →
- With for example, entropy S = S (U, N, V, x)

$$dS = \left(\frac{\partial S}{\partial U}\right)_{N,V,x} dU + \left(\frac{\partial S}{\partial N}\right)_{U,V,x} dN + \left(\frac{\partial S}{\partial V}\right)_{U,N,x} dV + \left(\frac{\partial S}{\partial x}\right)_{U,N,V} dx$$
$$dS = \left(\frac{1}{T}\right) dU + \left(\frac{a}{T}\right) dN + \left(\frac{p}{T}\right) dV + \left(\frac{X}{T}\right) dx$$

• Let's talk about some additional important concepts, such as the Boltzmann factor, Principle of Equipartition of Energy, Chemical Potential and the Gibbs free energy.

Let me emphasize that we do care where particles might be. The problem is just very complex computationally.

Statistical Mechanics: Gas Molecules in a Box

• For a large system, consider the average behavior of the system, such as pressure, temperature, particle concentration and probability distributions (for example of the velocity).

 $P(n) = \frac{(number \ of \ systems \ in \ the \ ensemble \ in \ which \ n \ is \ found)}{(total \ number \ of \ systems)}$



Suppose we would like to know how often a particle is in the left half of each box. If one particle is in the box, N = 1, then there are two possible states, L or R. If the particle is in the left, then we denote for state L that n = 1 and for R, n = 0.

The probability P(n) for n in L \rightarrow P(0) = 0.5 and P(1) = 0.5, with total summing to 1 (necessary condition).

An ensemble of boxes divided into halfs

Gas Molecules in a Box

Molecule 1	Molecule 2 (etc.)	n	P(n; 2 in this case)
R	R	0	1⁄4
R	L	1	1/2
L	R	1	1/2
L	L	2	1⁄4

Let's formazlie the discussion: each box has two volumes v and v' so that total volume V = v + v'.

The probability that a single particle is in volume v(v') is called p(q). Necessarily p + q = 1. From these two equations we find that p = v/V and q = v'/V. The total number N of particles can be distributed so that n is in v and n' = N - n is in v'.

$$P(n; N) = P(n; N, p) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

In[22]:= DiscretePlot[Table[PDF[BinomialDistribution[12, p], k], {p, {0.1, 0.5, 0.7}}] // Evaluate,





Out[24]= 2

•••• General: Exp[-6050.] is too small to represent as a normalized machine number; precision may be lost.

•••• General: Exp[-5000.] is too small to represent as a normalized machine number; precision may be lost.

.... General: Exp[-4050.] is too small to represent as a normalized machine number; precision may be lost.

···· General: Further output of General::munfl will be suppressed during this calculation.

- Binomial distribution for n = 12, with probabilities of 0.1, 0.5 or 0.7 for example.
- Comparison to a Gaussian disbribution
- Converted to a number, for example
- P(n=0,N=10)=0.001
- P(n=5,N=10)=0.246
- For our case of 12 boxes, the probability that all particles are in the left half is (1/2)¹² = 0.000244.
- Sampling at 1 million times a second gives a measurement that is 244 seconds long.
- Hence as the number of boxes increases the calculation time increases exponentially!

The Energy of a System: First Law of Thermodynamics

- What we just saw was how to consider the energy state of a system of particles, for example particles in 12 boxes.
- Classically we consider two ways the total energy of a system can change. Work W can be done on the system by the surroundings and heat Q can flow from the surroundings to the system. If no work is done, then $\Delta U = Q$. The work done by the system on surroundings is $\Delta U = -W$.

$$\Delta U = Q - W$$
 First Law of Thermodynamics



Fig. 3.6 No work is done on the system, but heat is added. The positions of the levels do not change; their average population does change



Fig. 3.7 Work is done on the system, but no heat flows. Each level has been shifted to a higher energy



FIg. 3.8 Symbols used to indicate various types of isolation in a system. **a** This system is completely isolated. **b** There is no heat flow through the double wall, but work can be done (symbolized by a piston). **c** No work can be done, but there can be heat flow through the single wall

Ensembles and the Basic Postulates

- The microstate of a system is determined by specifying the quantum numbers of each particle in the system. The total number of quantum numbers is the number of degrees of freedom.
- The microstate of a system is determined by all of the external parameters, such as volume, fields, on which the positions of the energy levels depend, and the total energy of the system, U. $P(of being in microstate i) = \frac{number of systems in microstate i}{total number of systems in the ensemble}$

Postulates:

- If an isolated system (really, an ensemble of isolated systems) is found with equal probability in each one of its accessible microstates, it is in equilibrium. Conversely, if it is in equilibrium, it is found with equal probability in each one of its accessible microstates.
- If it is not in equilibrium, it tends to change with time until it is in equilibrium. Therefore, equilibrium state is the most random, most probable state.

Α Α΄ Ν Ν΄ V, V', U΄ U

- Consider an isolated system (from the universe) that can exchange heat Q, work W, particles N within itself with a total energy U* (each of these terms is the sum of the components in the prime and unprimed system). The total number of microstates accessible to the total system is Ω*=Ω*_{tot}. Make a partition so that the total system comprises of A and A' sub-systems with the condition that particles cannot be exchanged. Then the total number of microstates in each partition depends on the energetics in the partition, Ω*(U).
- The probability of finding sytem A with energy U is

Thermal Equilibrium

$$P(U) = \frac{\Omega^*(U)}{\Omega^*_{tot}} = C\Omega^*(U)$$

- Now recall that when thinking of probabilities for example for isolated events, we can multiply each isolated event probability to obtain the total probability.
- That means the number of microstates for the total system is the product of the number for each subsystem:

$$\Omega^*(U) = \Omega(U)\Omega'(U')$$



A concrete example

U* = 10u.

System A		System A'		System A*
U	Ω	U'	Ω'	Ω*
2u	1	8u	7	7
3u	2	7u	6	12
4u	3	6u	5	15
5u	4	5u	4	16
6u	5	4u	3	15
7u	6	3u	2	12
8u	7	2u	1	7
				Ω* _{tot} =84

A concrete example



• The two terms are equal in equilibrium. Defining $\frac{1}{\tau} \equiv \frac{1}{\Omega} \frac{d\Omega}{dU}$.

• Notably, the constant parameter is defined so that $\tau = k_B^T$ with units and value 1.380*10⁻²³ J/K or 0.861*10⁻⁴ eV/K. This is the Boltzmann's constant.

Nernst Equation

- See handwritten notes, for the Nernst Equation, the potential difference depends on the concentration ratio on either side of the membrane. This form is an example of the Boltzmann factor.
- Depends on Faraday constant = 96485.32 C/mol, temperature T, gas constant R = 8.314 /mol*K

$$v_2 - v_1 = \frac{RT}{zF} ln(\frac{C_1}{C_2})$$

Chemical Potential

Consider two systems that exchange heat

$$\frac{1}{\Omega} \left(\frac{\partial \Omega}{\partial U} \right)_{\{N, V\}} = \frac{1}{\Omega'} \left(\frac{\partial \Omega'}{\partial U'} \right)_{N', V'}$$

• Or

$$\left(\frac{\partial S}{\partial U}\right)_{\{N, V\}} = \left(\frac{\partial S}{\partial U'}\right)_{N', V}$$

• The total number is constant.

. We define
$$\mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V}$$

- Equilibrium exists at T=T' and $\mu = \mu'$. Thus, two systems are in thermal and diffusive equilibrium when they have the same temperature and the same chemical potential.
- The units are in J/K

When a system out of equilibrium exchanges with a system in equilibrium

$$\Delta S^* = \left(\frac{\partial S^*}{\partial N}\right) \Delta N = \left(\frac{\partial S}{\partial N}\right) \Delta N + \left(\frac{\partial S'}{\partial N'}\right) \Delta N'$$

This can be rewritten to

$$\Delta S^* = \left(\frac{\mu' - \mu}{T}\right) \Delta N$$

If particles move from a system of higher chemical potential to one of lower chemical potential, the entropy of the total system increases.

The Chemical Potential of a Solution

- Consider a binary solution of solute and solvent and what changes to chemical potential occur.
- $S_{solution} S_{purewater, pure solute} = k_B ln(\Omega_{solution} / \Omega_{purewater, pure solute}) = k_B ln((N_w + N_s)! / N_w! N_s!) = k_B [ln(N_w + N_s)! ln(N_w!) ln(N_s!)]$
- Stirling's approximation gives

 $S_{solution} - S_{purewater, puresolute} = k_B[(N_w + N_s)ln(N_w + N_s) - N_w lnN_w - N_s lnN_s]$

• In terms of Gibbs free energy,

$$\Delta G = -T\Delta S = k_B T [N_w ln(\frac{N_w}{N_w + N_s} + N_s ln\frac{N_s}{N_w + N_s})]$$

• The terms in the natural log are the molecular fractions.

The Chemical Potential of a Solution

• For two molecular species, the energy (a fundamental equation) is $TdS = dU - \mu_w dN_w - \mu_s dN_s + pdV$

•
$$\Delta \mu_s = k_B T ln(C_2/C_1); \Delta \mu_w = -k_B T \Delta C_s/C_w$$

Gibbs Free Energy



• Consider glucose $C_6H_{12}O_8 + 6O_2 \rightarrow 6CO_2 + 6H_2O$ (aA + bB \rightarrow cC + dD)

 $G \equiv U - TS + pV$

• With U, V, N_A , N_B , N_C , N_D .

$$dG = dU - TdS - SdT + pdV + Vdp$$

$$TdS = dU - \mu_A dN_A - \mu_B dN_B - \mu_C dN_C - \mu_D dN_D + pdV$$

$$dG = \mu_A dN_A + \mu_B dN_B + \mu_C dN_C + \mu_D dN_D$$

$$taking \quad \mu_A = \mu_{A0} + k_B T ln \frac{C_A}{C_0}$$

$$dG = \left[c\mu_{C0} + d\mu_{D0} + a\mu_{A0} + b\mu_{B0} + k_B T ln \left(\frac{[C]^c[D]^d}{[A]^a[B]^b} \right) - k_B T ln \left(\frac{[C_0]^c[C_0]^d}{[C_0]^a[C_0]^b} \right) \right] dN$$

$$0 = \Delta G^0 + RT ln \left(\frac{[C]^c[D]^d}{[A]^a[B]^b} \right) = \Delta G^0 + RT lnK_{eq}$$

Then $\Delta G^0 = -680$ kcal, and for 36 ADP + 36 Pi \rightarrow 36 ATP + 36H20, $\Delta G^0 = +263$ kcal.

McDonald's Foods

Meal Menu	Energy (Kcal)	Protein (g)	Fat (g)	Saturated Fats (g)	Trans Fats (g)	Carbonates (g)	Sugar (g)	Na (mg)
Hamburger	267	13	9	4.4	0	33	5	340
lce cream cone	147	3	5	2.3	0	24	18	70
Coca cola	199	0	0	0	0	50	50	10