MODERN PHYSICS

OPTICS

Optics

- Some topics we will cover

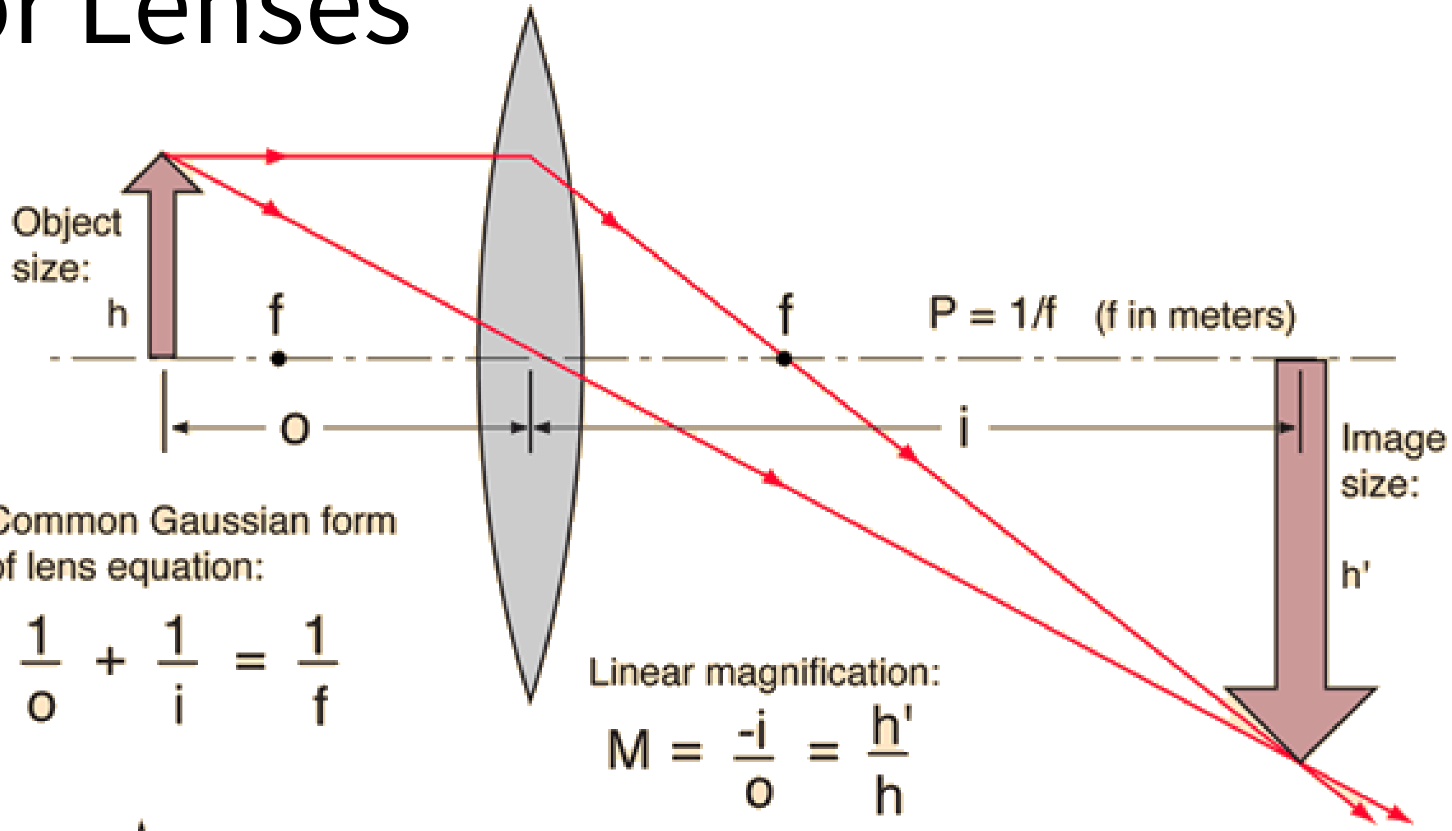
- Ray tracing
- Refractive index
- Waves
- Photoelectric Effect
- Reflectance
- Compton Effect
- E&M
- Polarization

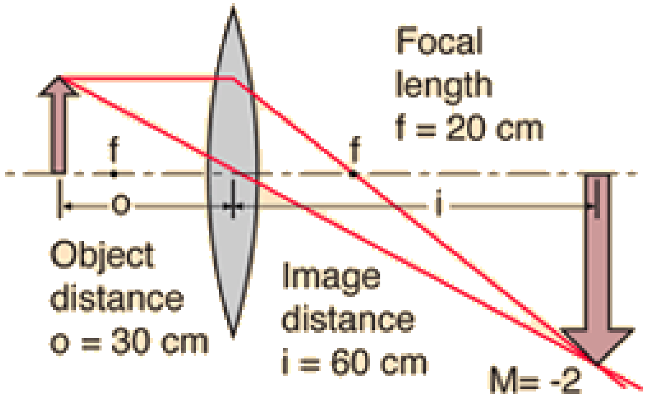
Ray Diagrams for Lenses

- For a convex lens, rays for example from n = 1 material enter "slower" material with n > 1 (the lens). This bends light toward the normal.
- Rays entering "faster" material bend away from normal.

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

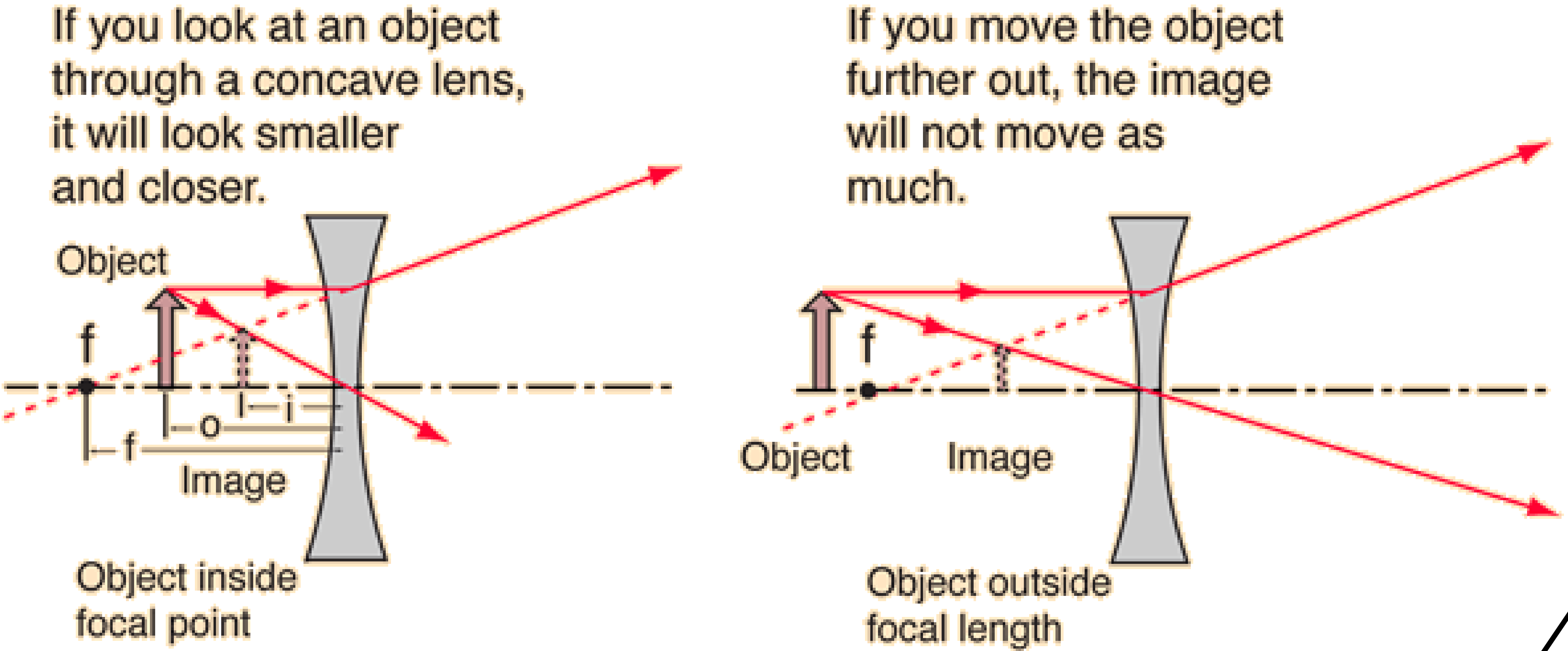
$$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{\text{focal length}}$$





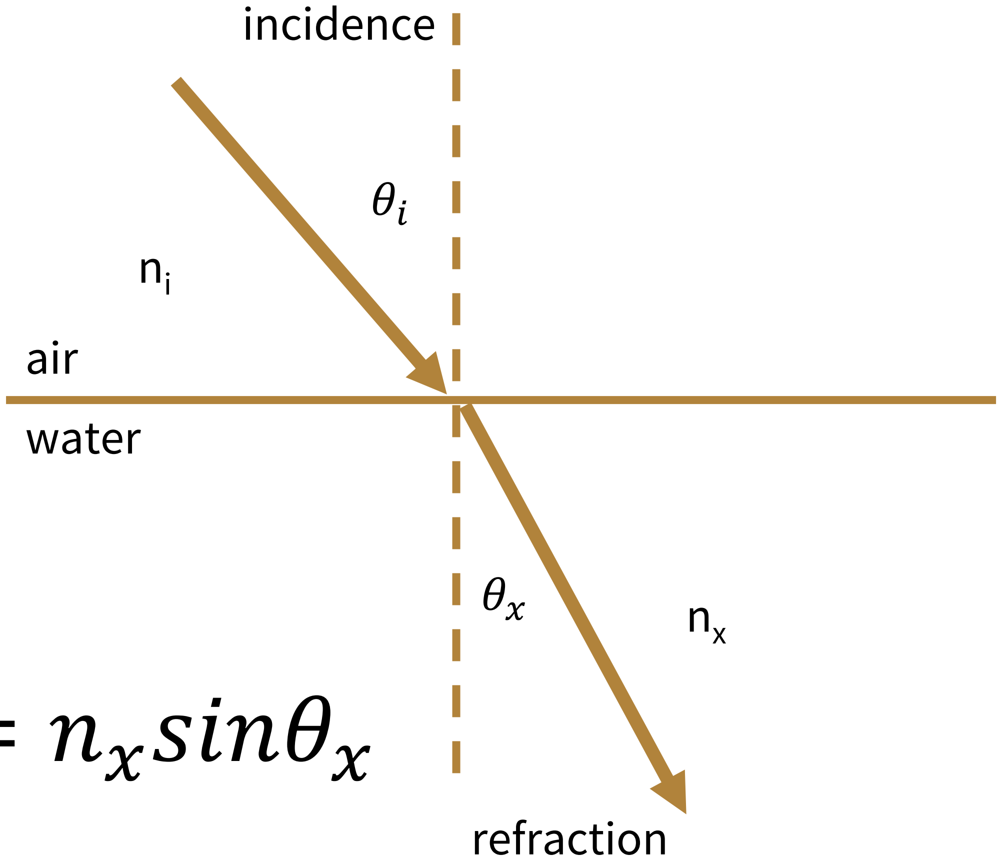
Ray Diagrams for Lenses

- For a concave lens



Snell's Law

- Describes the path of a beam of light at the interface between two materials with differing refractive index – a material property describing the change in speed when it changes medium.



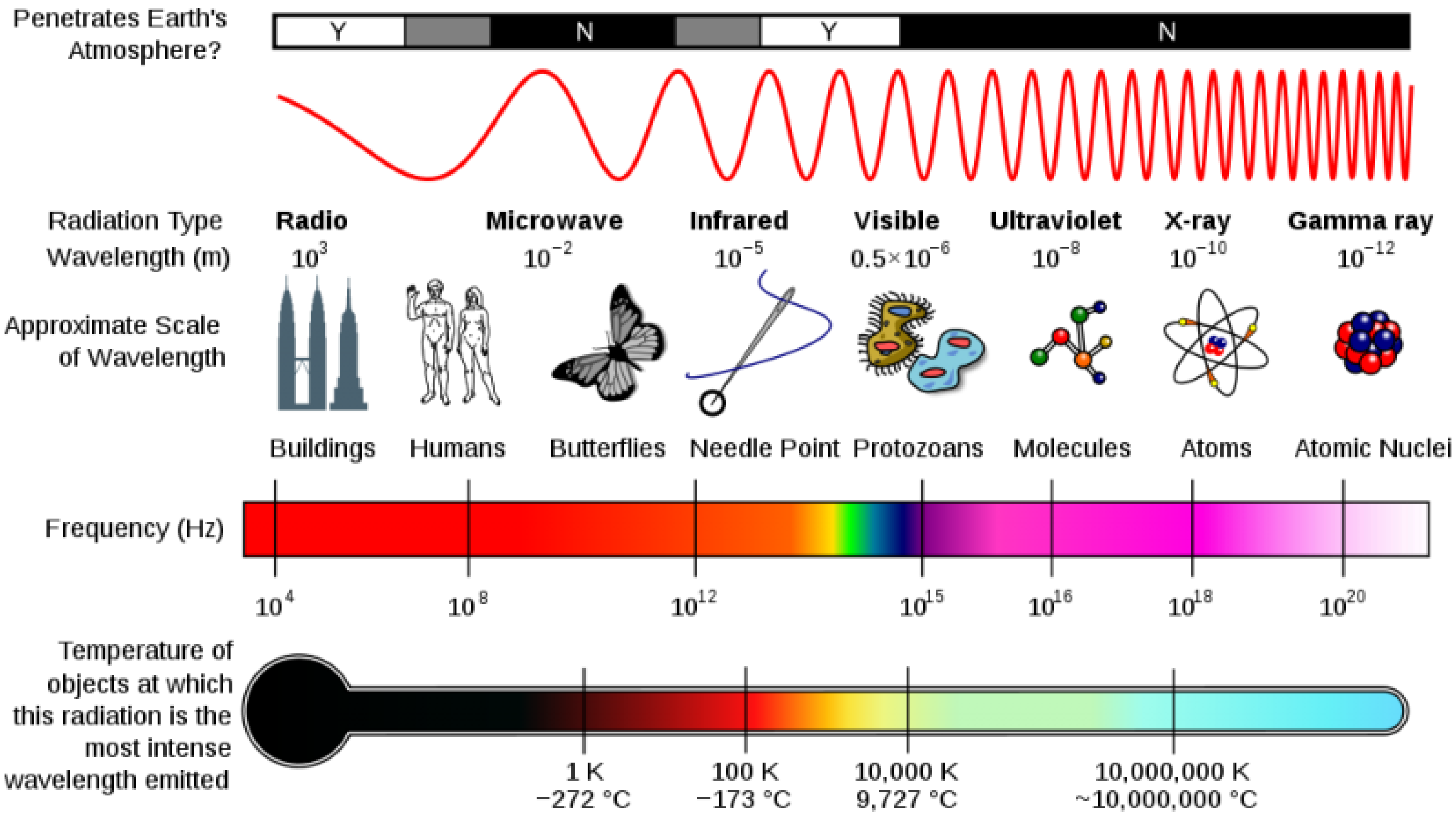
$$n_i sin\theta_i = n_x sin\theta_x$$

Material	Index of Refraction
Vacuum	1.0000
Air	1.0003
Ice	1.31
Water	1.333
Ethyl Alcohol	1.36
Plexiglass	1.51
Crown Glass	1.52
Light Flint Glass	1.58
Dense Flint	1.66
Glass Zircon	1.923
Diamond	2.417
Rutile	2.907
GaP	3.50

Electromagnetic Waves

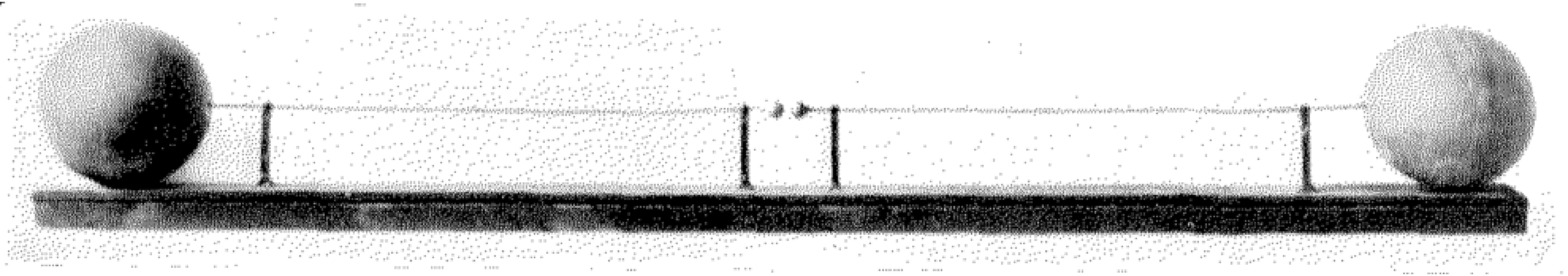
- James Maxwell derived formulaic understanding of electromagnetism:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$



Photoelectric Effect

- Or waves behaving as particles.
- Historically, this effect was investigated when Heinrich Hertz shown ultraviolet light on metal balls and noticed stronger sparks in the air gap.



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- Interestingly, the sparks, not thoroughly investigated by Hertz, meant that there’s more than meets the eye, catastrophe!

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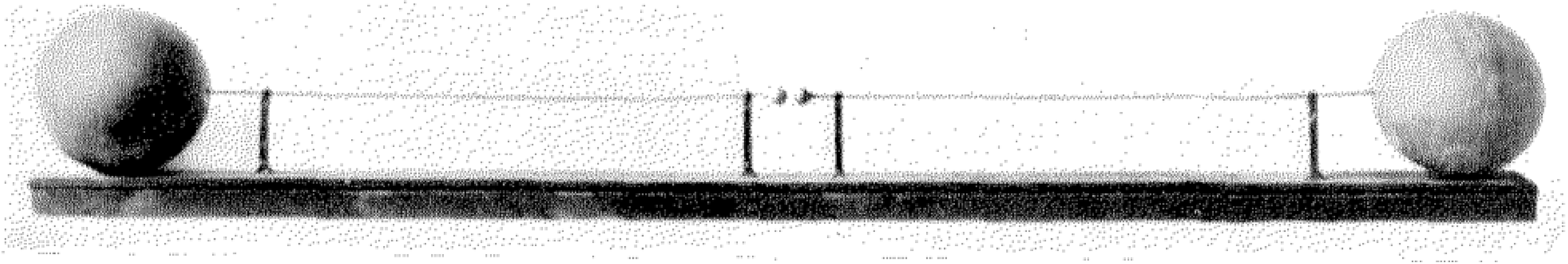

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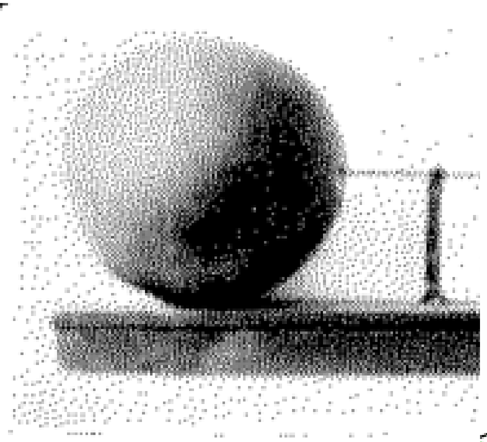

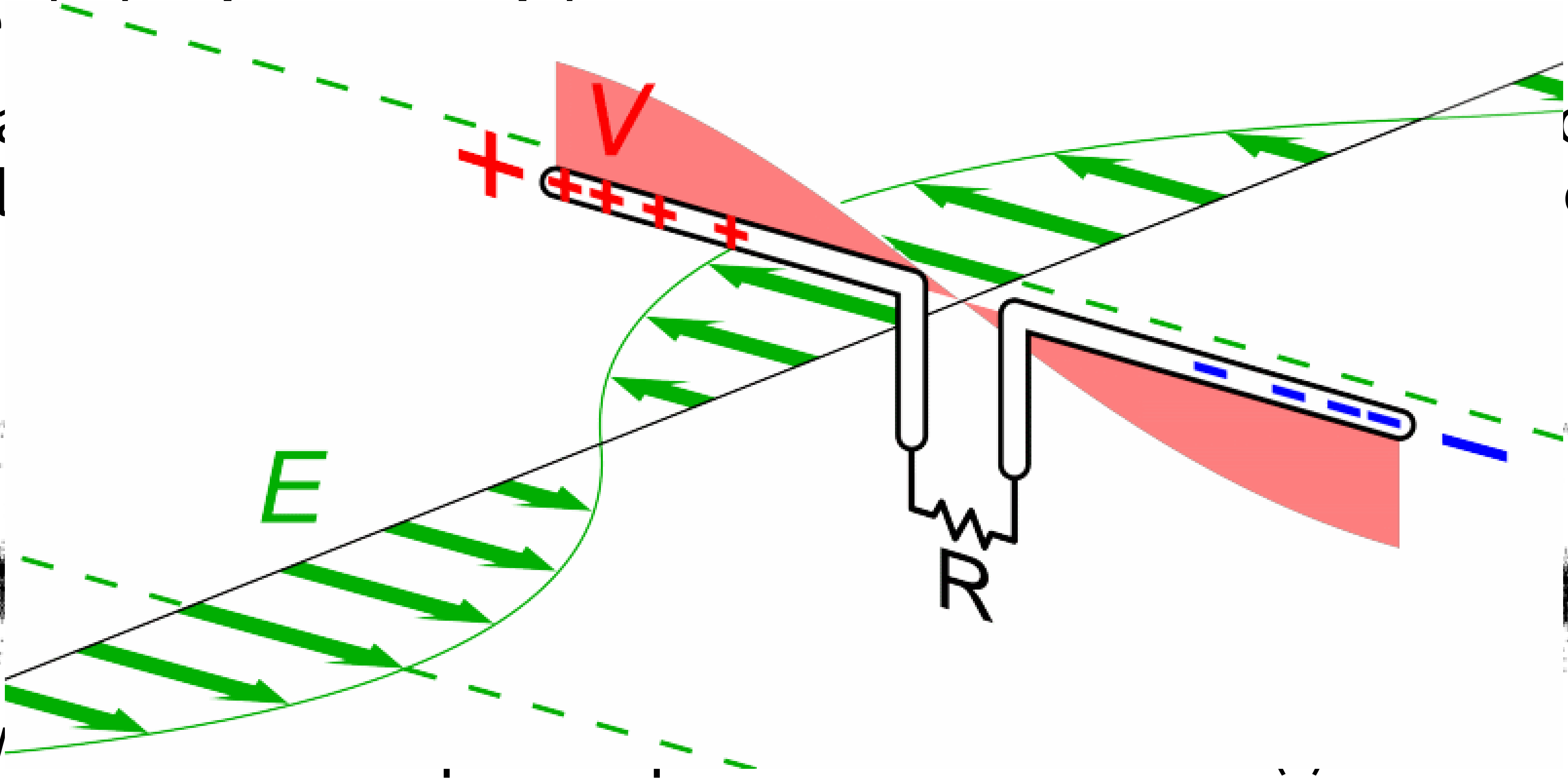

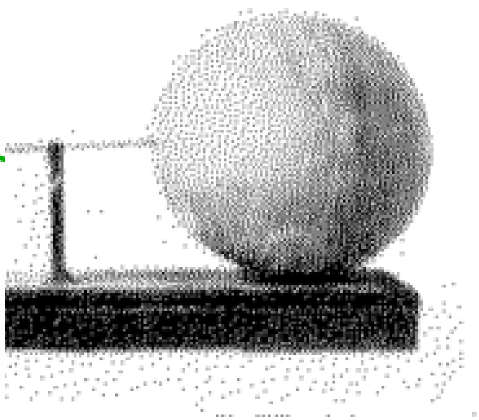

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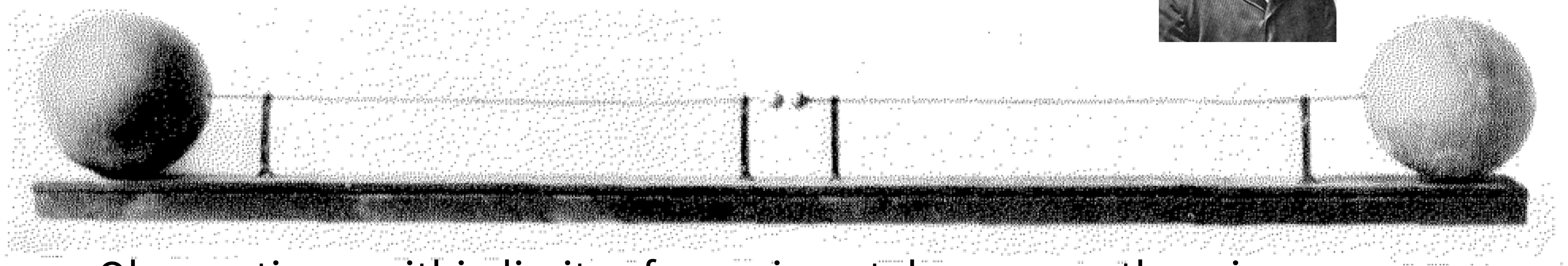
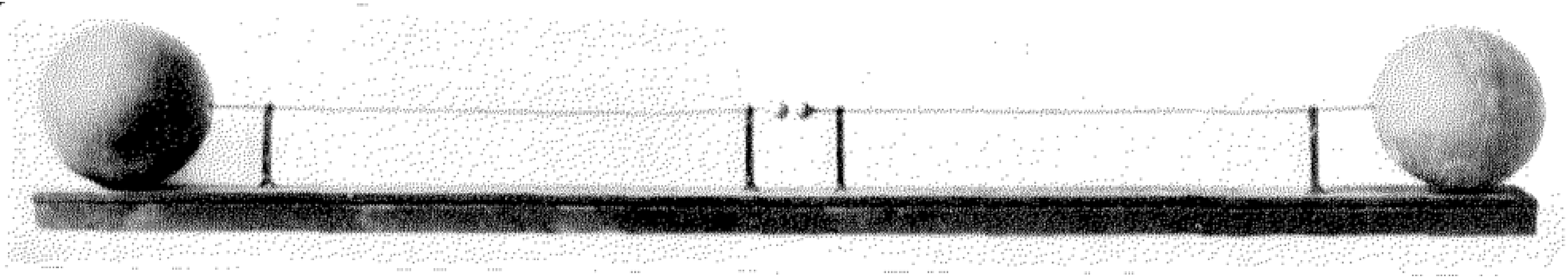
Photoelectric Effect



- Observations: within limits of experimental accuracy, there is no time interval between the arrival of light at a metal surface and the emission of photoelectrons.
- A bright light yields more photoelectrons than a dim one of the same frequency, but the electron energies remain the same.
- The higher the frequency of light, the more energy the photoelectrons have. Below a certain value of frequency, no electron emission occurs.

Photoelectric Effect

- Einstein realized that the photoelectric effect could be understood if the energy in light is not spread out over wavefronts but concentrated in small packets, or photons, thus solving the “UV catastrophe.”

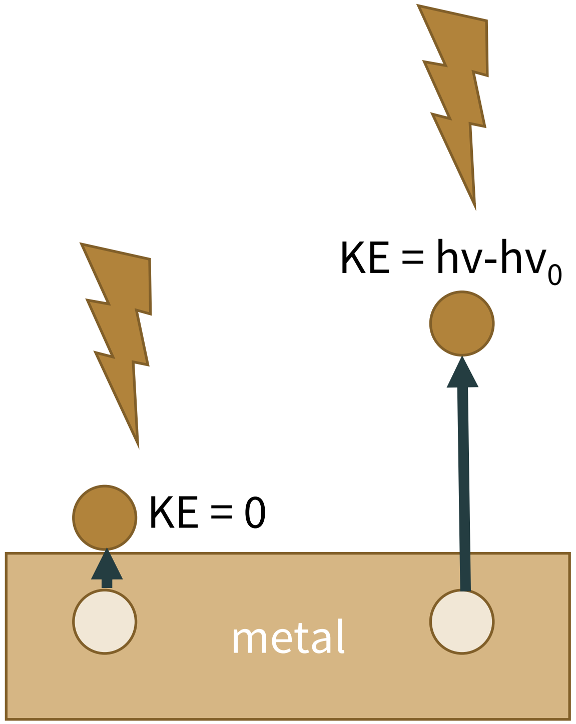


$$h\nu = KE_{max} + \phi; \phi = h\nu_0$$

Photoelectric Effect – An example

$$h\nu = KE_{max} + \phi; \phi = h\nu_0$$

- Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m^2 is directed at a potassium surface.
- The maximum kinetic energy of the photoelectrons is
- $E_p = \frac{1.24x10^{-6} eVm}{350\ nm \frac{10^{-9} m}{nm}} = 3.5\ eV. KE_{max} = h\nu - \phi = 3.5\ eV - 2.2\ eV = 1.3\ eV$



- The photon energy in joules is 5.68*10^{-19} J. The number of photons that reach the surface per second is
- $n_p = \frac{\frac{E}{t}}{E_p} = \frac{P}{A} * \frac{A}{E_p} = \frac{\left(1.00\frac{W}{m^2}\right)\left(1.00*10^{-4}\,m^2\right)}{5.68*10^{-19}\frac{J}{photon}} = 1.76 * 10^{14}\frac{photon}{s}$.
- If 0.50% of the incident photons produce photoelectrons, then the rate at which photoelectrons are emitted is 8.8*10^{11} photoelectrons/s.

Photoelectric Effect – Some Materials

Metal	Work function (eV)
Cesium	1.9
Potassium	2.2
Sodium	2.28-2.3
Lithium	2.5
Calcium	3.2
Copper	4.7
Silver	4.7
Platinum	6.4
Aluminum	4.08
Cobalt	3.90
Zinc	4.31
Lead	4.14

Example

- Consider a potassium surface that is 75 cm away from a 100 W bulb. Suppose that the energy radiated by the bulb is 5% of the input power. Treating each K atom as a circular disc of diameter 1 Å, determine the time required for each atom to absorb an amount of energy equal to its work function of 2.0-2.2 eV, according to the wave interpretation of light.

Treating the bulb as a point source, the intensity at the location of the potassium surface is

$$intensity = \frac{power}{area\ of\ sphere} = \frac{100\ W * 0.05}{4\pi(0.75\ m)^2} = 0.707\ W/m^2$$

The number of photons that reach the surface per second is

$$power\ per\ atom = intensity\ * (area\ per\ atom) = \frac{\frac{0.707W}{m^2}\pi(1 * 10^{-10}m)^2}{4} = 5.56 * 10^{-21}m$$

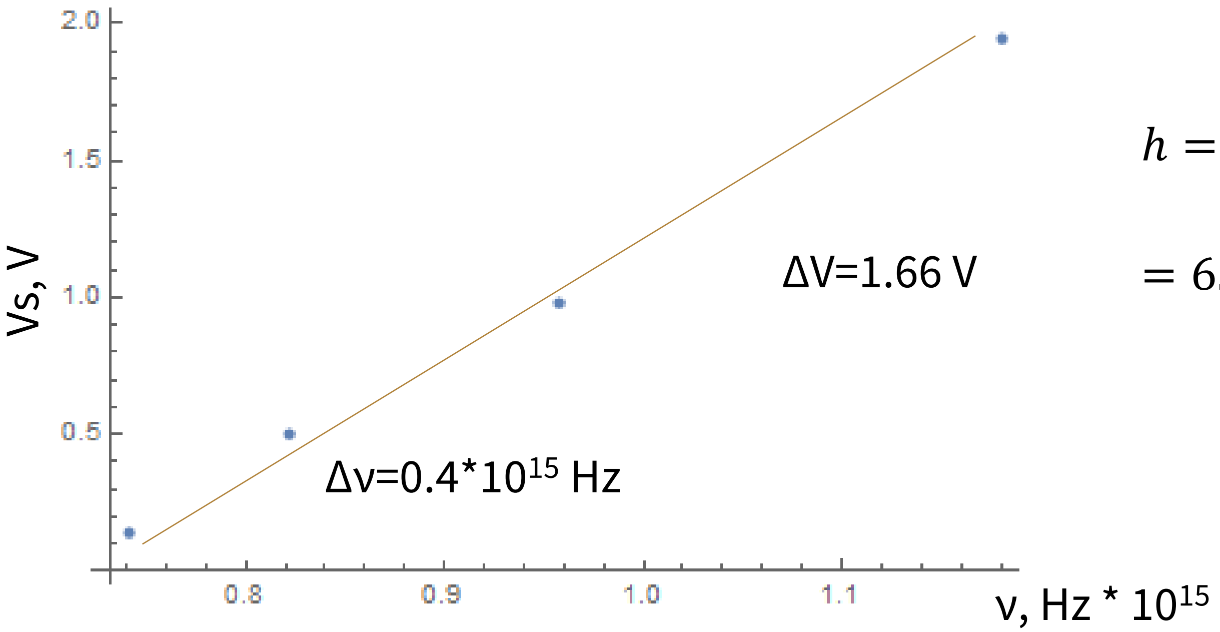
The time interval to absorb 2.0 eV is then power =energy/time or $t = 2.2eV * 1.6 * \frac{\frac{10^{-19}J}{eV}}{5.56*\frac{10^{-21}J}{s}}$ =63.094 s

Note that in this calculation assumed all the incident energy is absorbed. In practice the time is longer.

Measuring Planck's Constant With the Photoelectric Effect

- Here's an example of measuring the photoelectric effect using a calcium emitter:

λ, Å	2536	3132	3650	4047
ν, Hz * 10^{15}	1.18	0.958	0.822	0.741
Vs, V	1.95	0.98	0.50	0.14



$$h = e(slope) = 1.6 * 10^{-19} * \frac{1.66}{0.4 * \frac{10^{15}}{s}}$$

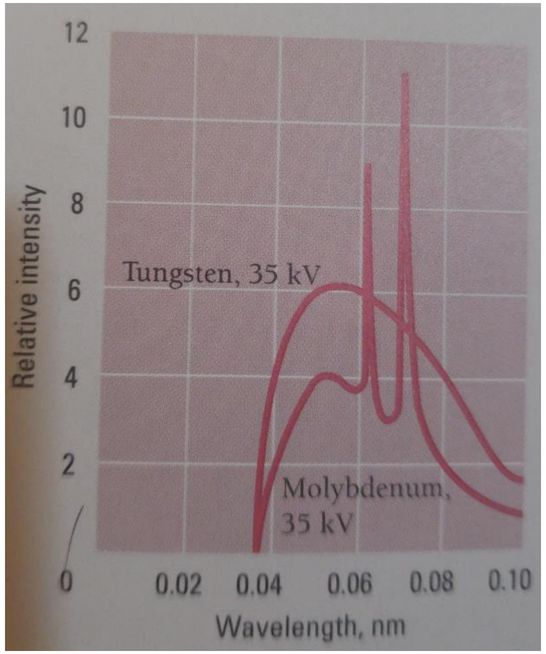
$$= 6.6 * 10^{-34} J/s$$

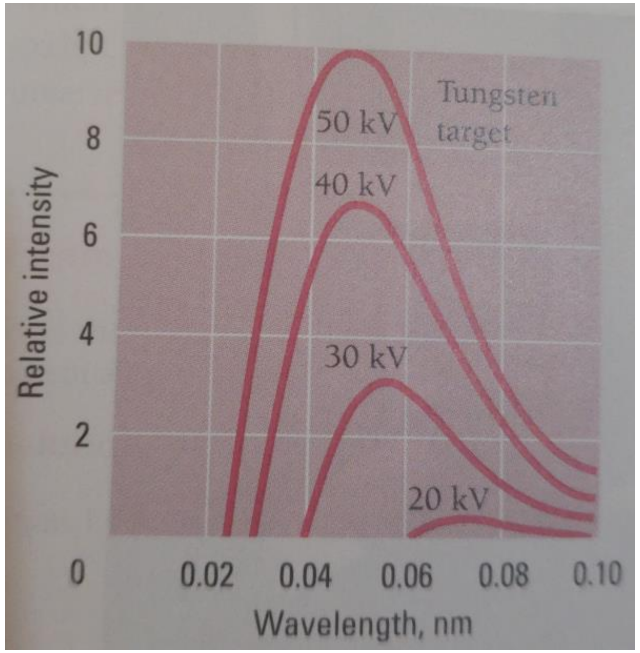
X-rays and the Compton Effect - Revisited

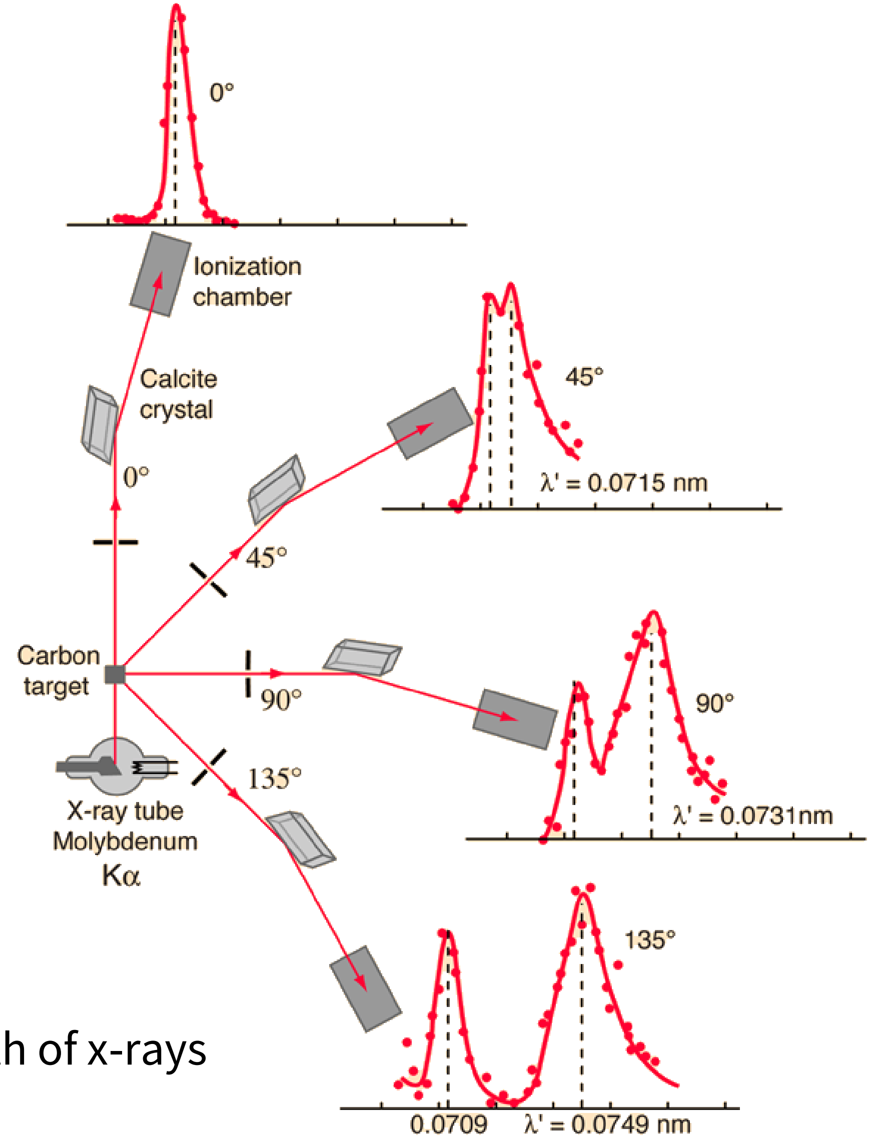
- A short video, Roentgen and the discovery of the X-rays [https://www.youtube.com/watch?v=HRNm9QWkDyY].
- Production of X-rays [https://www.youtube.com/watch?v=T1WwHh4b__M&list=PLwSi3mH0cLbojFSbkkPGfx8QfzFtyVURq&index=3&t=0s].

X-rays and the Compton Effect - Revisited

For discussion on Tuesday 29th



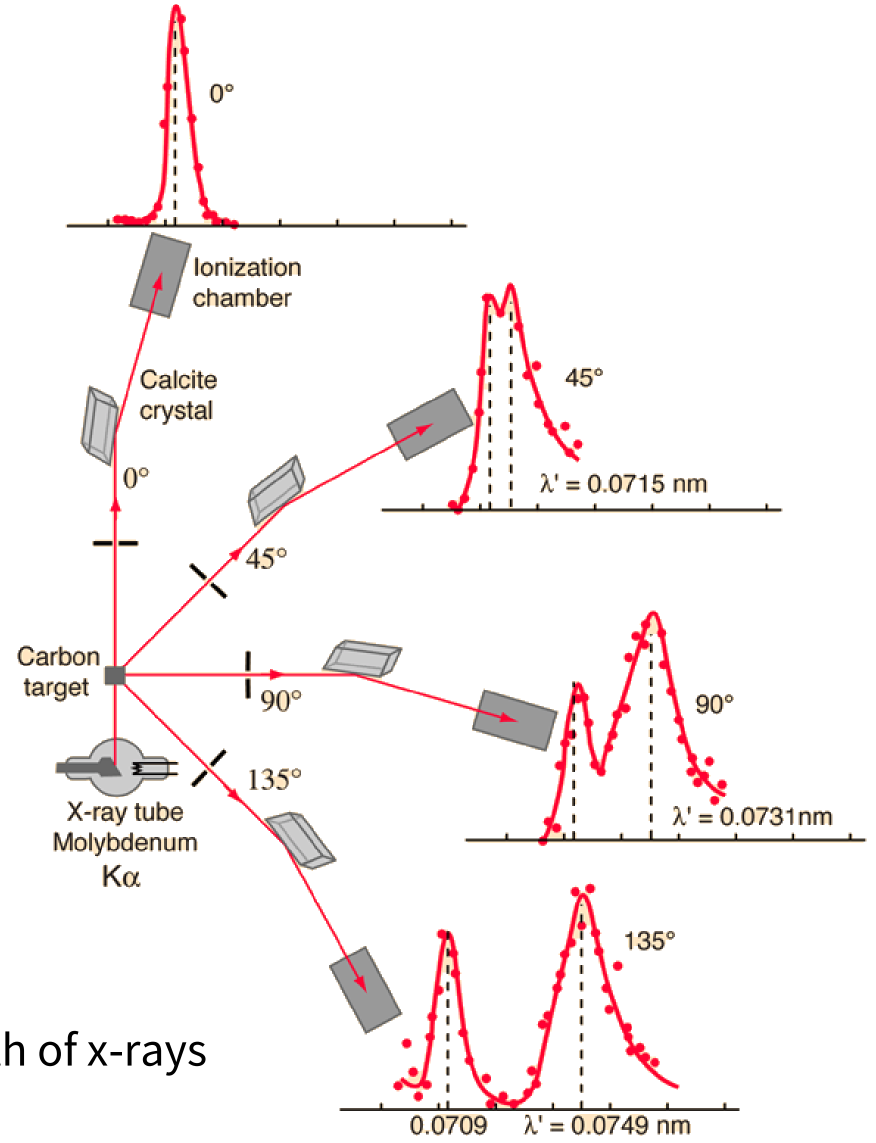




0.0709 nm corresponds to wavelength of x-rays from Molybdenum K-alpha source

X-rays and the Compton Effect - Revisited

- Other x-ray sources include: synchrotron (from accelerating charged particles to relativistic speeds with energies 2 to 8 GeV and peaking at wavelengths 4 to 0.5 Å)



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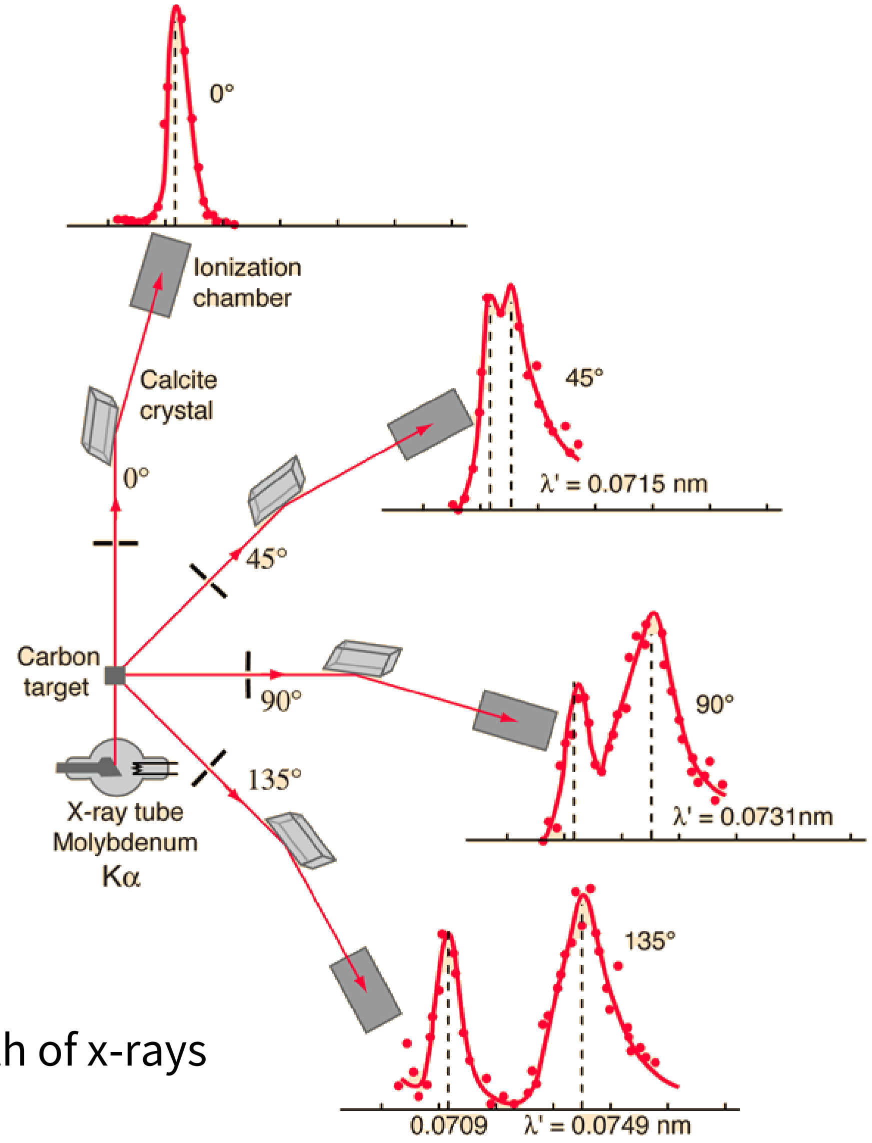
X-rays and the Compton Effect - Revisited

- X-rays typically 0.01 to 10 nm.
- Medical uses of x-rays immediately apparent.
- More sophisticated equipment such as CAT (computed tomography) scanner produce cross sectional images of body.



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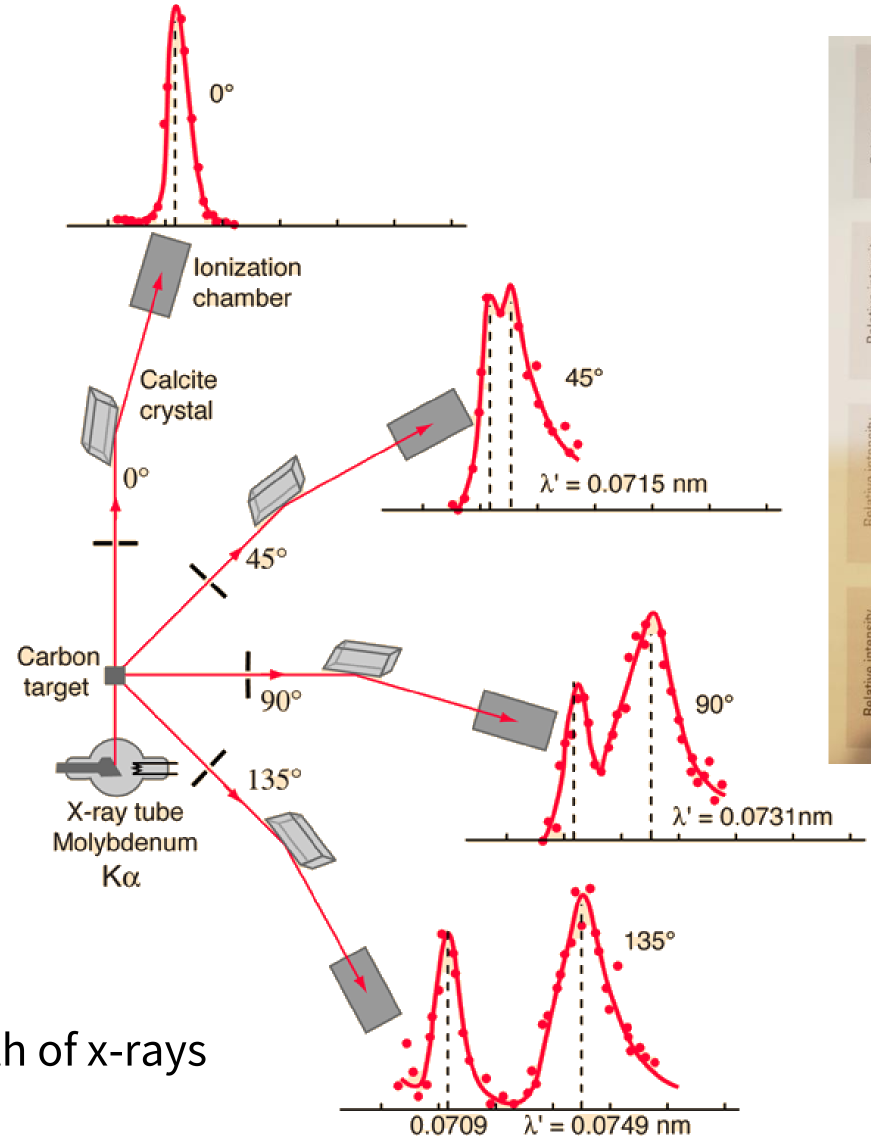

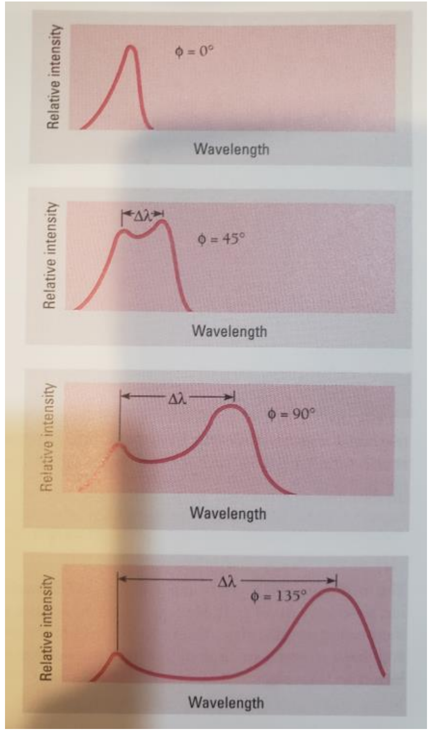

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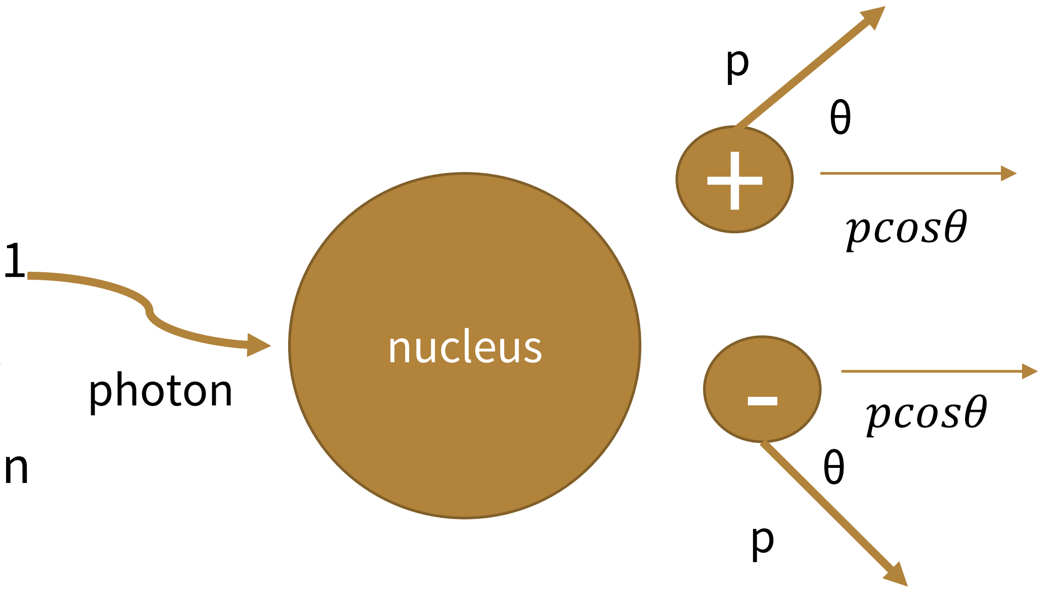
Pair Production or Energy Into Matter

- $Q_{e-} + Q_{e+} = 0$
- $e^+ + e^- \rightarrow \gamma + \gamma$
- The rest energy of an electron or positron is $m_o c^2 = 0.51$ MeV, so pair production requires 1.02 MeV (with corresponding maximum photon wavelength 1.2 pm). These photons are called gamma rays.
- An example to analyze this occurrence: Pair production cannot occur in empty space.
- Let's imagine an electron-positron pair.
- $h\nu = 2mc^2 \; and \; \frac{h\nu}{c} = 2pcos\theta \; or \; h\nu = 2mc^2\left(\frac{v}{c}\right) cos\theta. Since \; \frac{v}{c} < 1 \; and \; cos\theta < 1, h\nu < 2mc^2$
- Something has to carry away part of the initial photon momentum.



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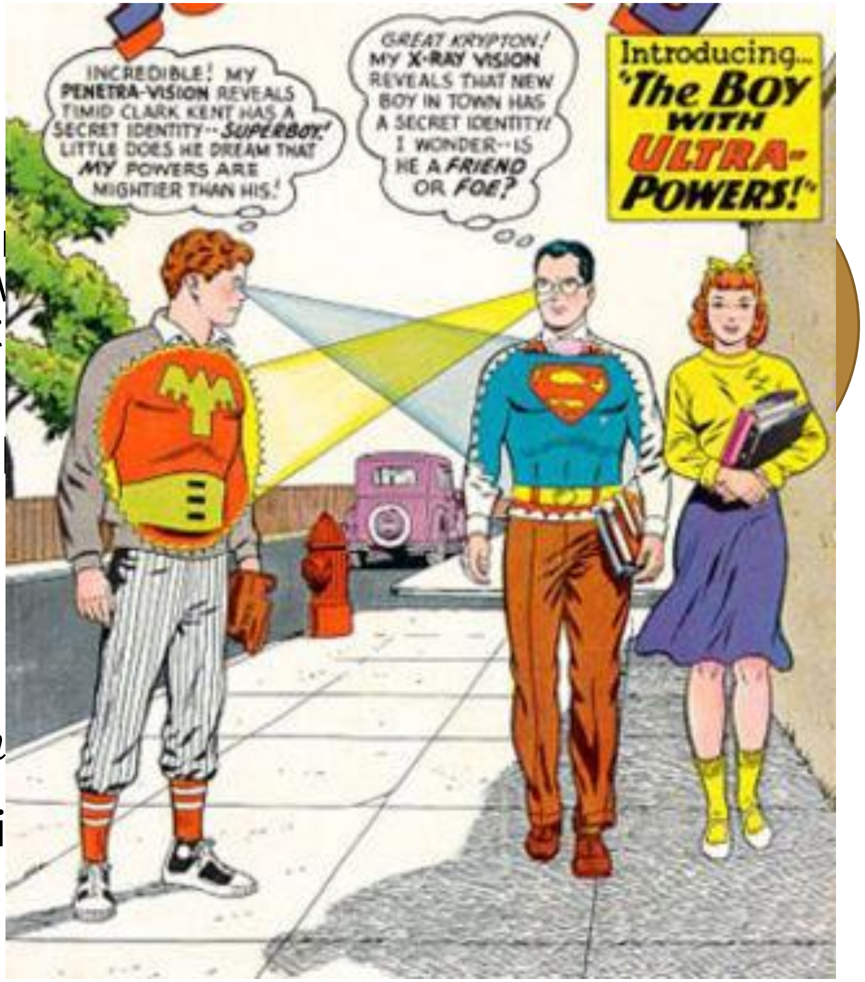
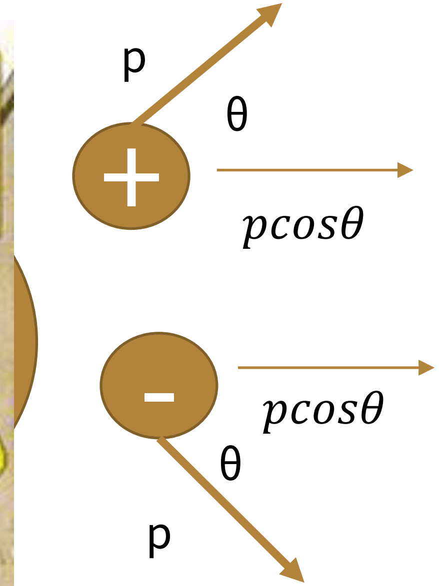


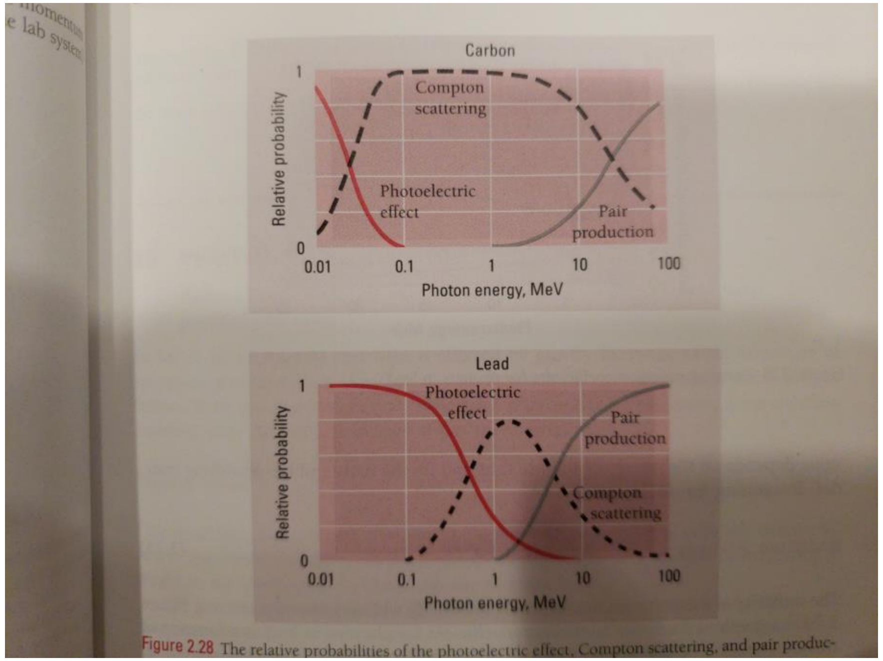


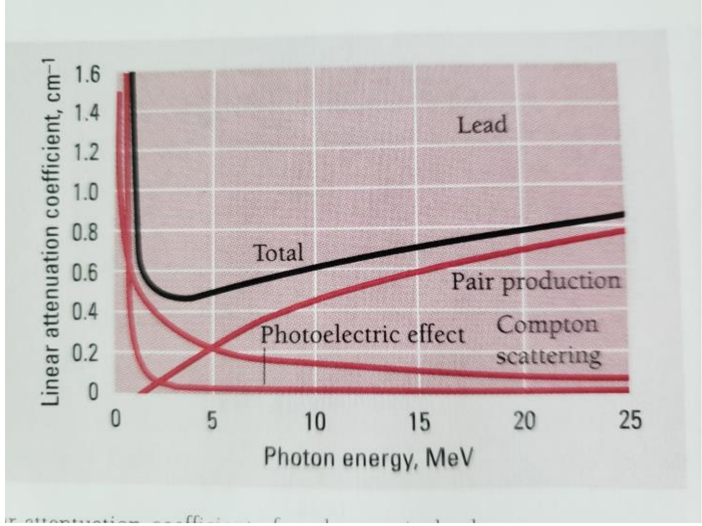
https://io9.gizmodo.com/everything-you-ever-wanted-to-know-about-supermans-eyeb-360939

Photon Absorption

- Photons: light, x-rays, gamma rays interact with light by
  - photoelectric effect, low energies
  - Compton scattering, 10s keV – 1 MeV
  - pair production, > 1.02 MeV
- The intensity of an x- or gamma-ray beam is equal to the rate at which it transports energy per unit cross-sectional area of the beam. So the fractional energy –dI/I lost by the beam in passing through a thickness dx of a certain absorber is found to be:

$$-\frac{dI}{I} = \mu dx \ \rightarrow I = I_0 e^{-\mu x}$$





Photon Absorption

- Example: The linear attenuation coefficient for a 2.0 MeV gamma ray in water is 4.9 /m. What is the relative intensity of a beam of 2.0 MeV gamma rays after it has passed through 10 cm of water and how far must such a beam travel in water before its intensity is reduced to 1 percent of its original value?
- $\mu x = (4.9\ m^{-1})(0.10\ m) = 0.49\ then\ \frac{I}{I_0} = e^{-0.49} = 0.61.$
- The intensity of the beam is reduced to 61 percent of its original value after passing through 10 cm or water.
- $\frac{I_0}{I} = 100, so\ that\ the\ general\ equation\ becomes\ \mu x = \ln\left(\frac{l_0}{l}\right) = \frac{lnI_0/I}{4.9\ m^{-1}}$=0.94 m.

Photons and Gravity $E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$; $p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$ $E = \sqrt{m_0^2 c^4 + p^2 c^2}$

- From our discussion of the energy mass equivalence, we see that photons while lacking rest mass, have gravitational mass (when moving). $p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow m = \frac{h\nu}{c^2}$

- Recall that if we dropped a stone with mass m from height H near the Earth's surface, the gravitational pull of the Earth accelerates the stone as it falls so that potential energy mgH is gained. The kinetic energy $1/2mv^2 = mgH$ at the end of the drop, meaning $v=(2gH)^{1/2}$.

Photons and Gravity

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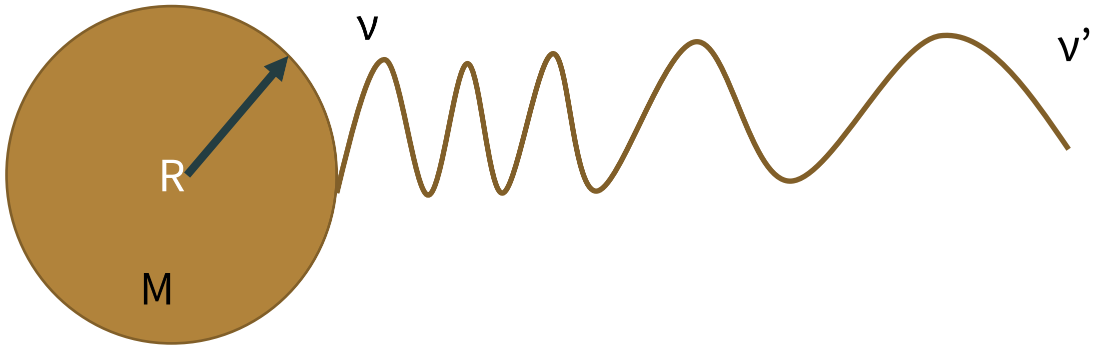


- And the Gravitational Red Shift:

$$PE = -\frac{GMm}{R} = -\frac{GMh\nu}{c^2 R} \; so \; E = h\nu - \frac{GMh\nu}{c^2 R}$$

- Farther away from star, the energy converts to electromagnetic energy $h\nu' = h\nu - \frac{GMh\nu}{c^2 R} \rightarrow \frac{\nu'}{\nu} = 1 - \frac{GM}{c^2 R} \rightarrow \frac{\Delta\nu}{\nu} = \frac{\nu' - \nu}{\nu} = 1 - \frac{\nu'}{\nu} = \frac{GM}{c^2 R}$

Photons and Gravity

- A reminder that the values of little g = 9.8 m/s^2 and big G = 6.67 × 10^{-11} Newtons kg^{-2} m^2.
- What if $\frac{GM}{c^2R}$ > 1? Astronomical objects that fall in this category are known as black holes,

due to their massive energy such that the stars would effectively be invisible. Photons cannot escape the gravitational pull.

- It turns out that the correct description is with $\frac{GM}{c^2R}$ > 0.5 so that the Schwarzschild radius is $\mathrm{R}_s = \frac{2GM}{c^2}$. The body is a black hole if all its mass is inside a sphere with this radius. The escape speed from a black hole is equal to the speed of light.
- We can detect black holes from its gravitational effect on nearby stars, for example in a double-star system. There are also significant x-ray emissions (from the strong gravitational compression and heating) that can be detected.
- An example is the Cygnus X-1 star that has mass 8 times our sun and a radius of only about 10 km. The region around the black hole that emits x-rays extends outward for several 100 km!

De Broglie Relation

- The quantum description of light. The idea is that the momentum of light can be described as packets with the following relation
- Classical $\mathrm{mv} \ = p = \frac{h\nu}{c} = \frac{h}{\lambda}$ quantum
- This is fundamental because recall discussion on waves:
- In water waves the quantity that varies is the height of water surface. In sound waves, it is pressure. In light waves, the electric and magnetic fields vary. In matter waves? It is the wavefunction,Ψ which describes a probability to find the matter there. What we physically observe, is the probability density, $|\psi|^2$.

De Broglie Waves

- We can solve for the wave equation as before, and include relativistic effects.

$$v_g = \frac{d\omega}{dk}$$

Angular frequency $$\omega = 2\pi\nu = \frac{2\pi mc^2}{h} = \frac{2\pi m_0 c^2}{h\sqrt{1 - \frac{v^2}{c^2}}}$$

Wave number $$k = \frac{2\pi}{\lambda} = \frac{2\pi m v}{h} = \frac{2\pi m_0 v}{h\sqrt{1 - \frac{v^2}{c^2}}}$$

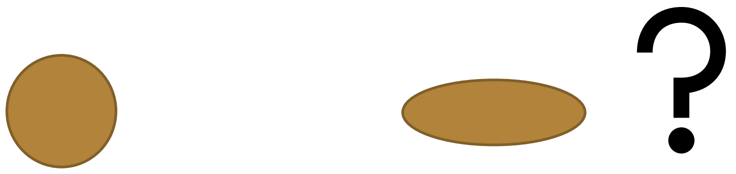
Phase velocity

$$v_p = \frac{\omega}{k}$$

De Broglie Waves – An example

- Find the de Broglie wavelengths of a 46 gram golfball with a velocity of 30 m/s and an electron with a velocity of 10^7 m/s.

$$v \ll c, we\ can\ assume\ m = m_0\ so\ that\ \lambda = \frac{h}{mv} = 6.63 * \frac{10^{-34} Js}{0.046kg * \frac{30m}{s}} = 4.8 * 10^{-34} m$$



$$v \ll c, so\ with\ m = m_0 = 9.11 * 10^{-31} kg\ we\ find\ \lambda = \frac{h}{mv} = 6.63 * \frac{10^{-34} Js}{9.11 * 10^{-31} * 10^7} = 7.3 * 10^{-11} m$$

De Broglie Waves – An example

- Find the kinetic energy of a proton whose de Broglie wavelength is 1.000 fm = 1.000 * 10^{-15} m, which is roughly the proton diameter.
- A relativistic calculation is required in this case, unless p*c for the proton is much smaller than the proton rest mass of E_0= 0.938 GeV.

$$pc = (mv)c = \frac{hc}{\lambda} = 4.136 * 10^{-15} eVs \; * \frac{\frac{3 * 10^8 m}{s}}{1 * 10^{-15}} - 1.240 * 10^9 eV = 1.241\ GeV$$

$$E = \sqrt{E_0^2 + p^2c^2} = \sqrt{0.938\ GeV^2 + 1.234\ GeV^2} = 1.555\ GeV$$

$$the\ corresponding\ KE = E - E_0 = 1.555 - 0.938\ GeV = 617\ MeV$$

What are de Broglie waves?

- In water waves the quantity that varies is the height of water surface. In sound waves, it is pressure. In light waves, the electric and magnetic fields vary. In matter waves? It is the wavefunction,Ψ which describes a probability to find the matter there. What we physically observe, is the probability density, $|\psi|^2$.
- This number and its magnitude describes the probability of existence.
- An important distinction: When an experiment is performed to detect electrons, for instance, a whole electron is either found at a certain time and place or it is not; there is for today's point of view "no" such thing as a 20% of an electron. However it is entirely possible for there to be a 20% chance that the electron be found at that time and place, with the likelihood specified by $|\psi|^2$.

Electromagnetic Waves

- The Maxwell's Equations and some examples

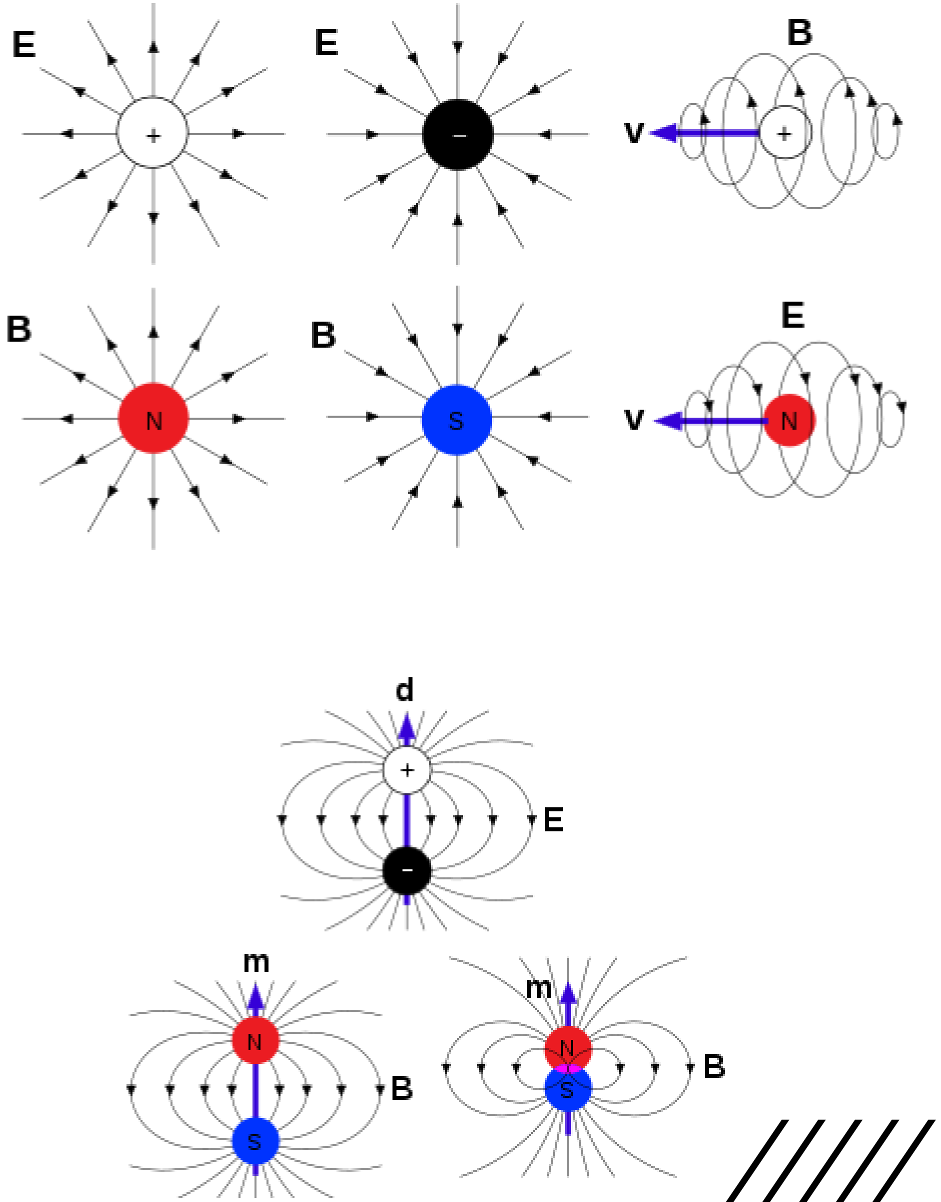
$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot B = 0 \qquad \nabla \cdot B = \rho_m$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \times E = -\frac{\partial B}{\partial t} + j_m$$

$$\nabla \times B = \mu_0 j + \frac{1}{c^2}\frac{\partial E}{\partial t} \qquad \nabla \times B = \mu_0 j + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

Describing magnetic monopoles



Classical Waves (recap)

- Group velocities
- And read about it in the text!

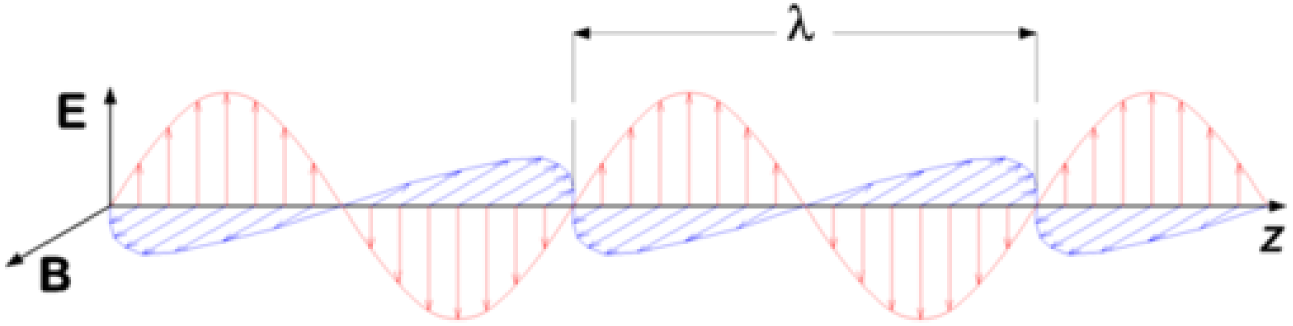
The Strangeness of Quantum Particles

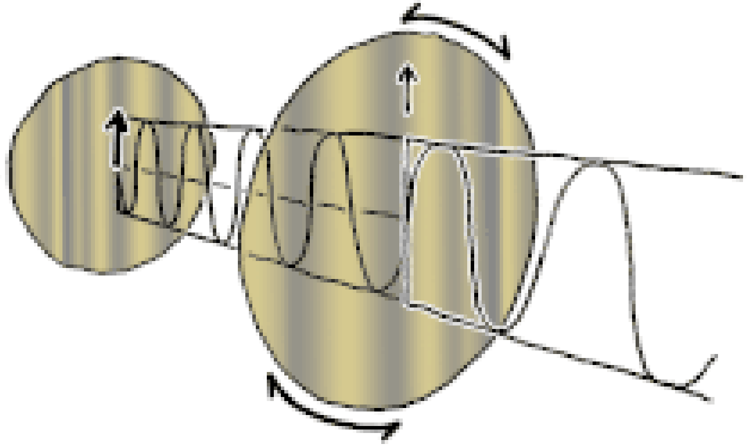
- Particle Diffraction
- Prelude to Quantum, putting a particle in a box:
- Uncertainty Principle in space and momentum
- Uncertainty Principle in energy and time

Gaussian Function

- A reminder, when a set of measurements is made of some quantity x in which the experimental errors are random, the result is often a Gaussian distribution
- $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$, here the standard distribution is $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - x_0)^2}$
- Note that the probability that a measurement is between range x_1 and x_2 is then $P_{x_1 x_2} = \int_{x_1}^{x_2} f(x)dx$
- Solving a function of the form of the probability above is much of what is done to understand quantum mechanical wave function, for example $\Psi(x) = \int_0^\infty g(k)f(k,x)dk$

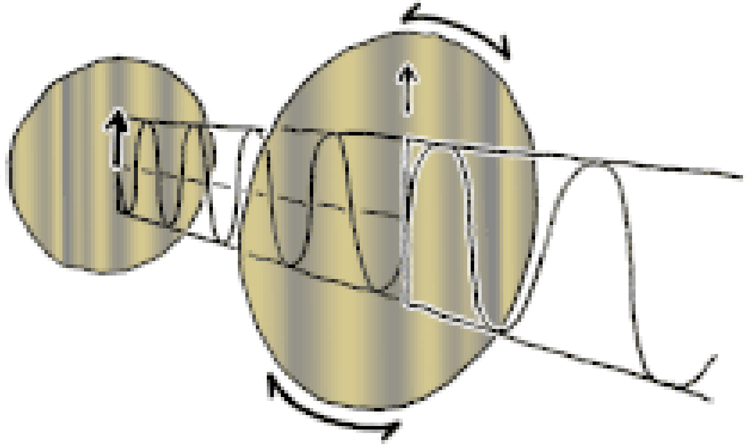
Polarization

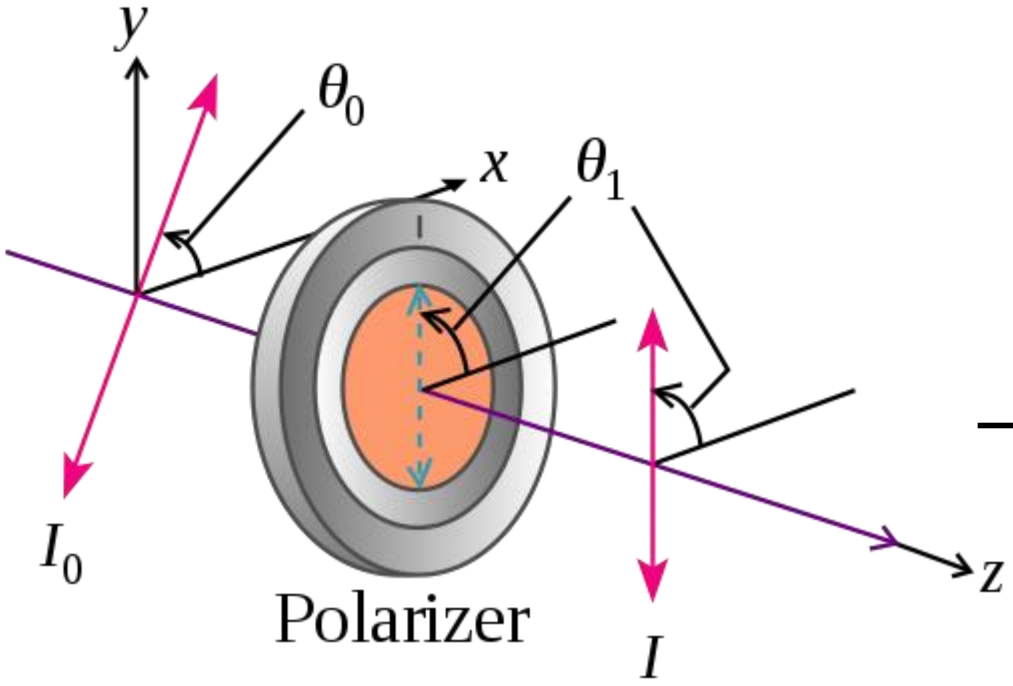




- The electromagnetic wave propagates orthogonal to the electric and magnetic wave components, with the energy described by the Poynting vector $S = E \times B/\mu_0$.
- The dot product is necessarily zero, $E \cdot B = 0$.

Polarization – Malus' Law

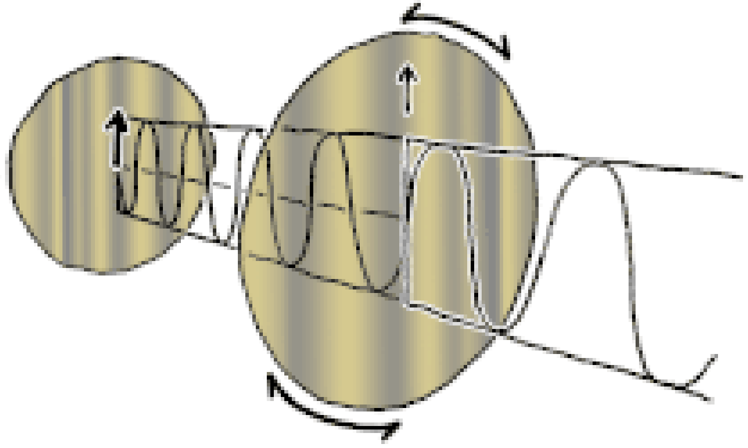


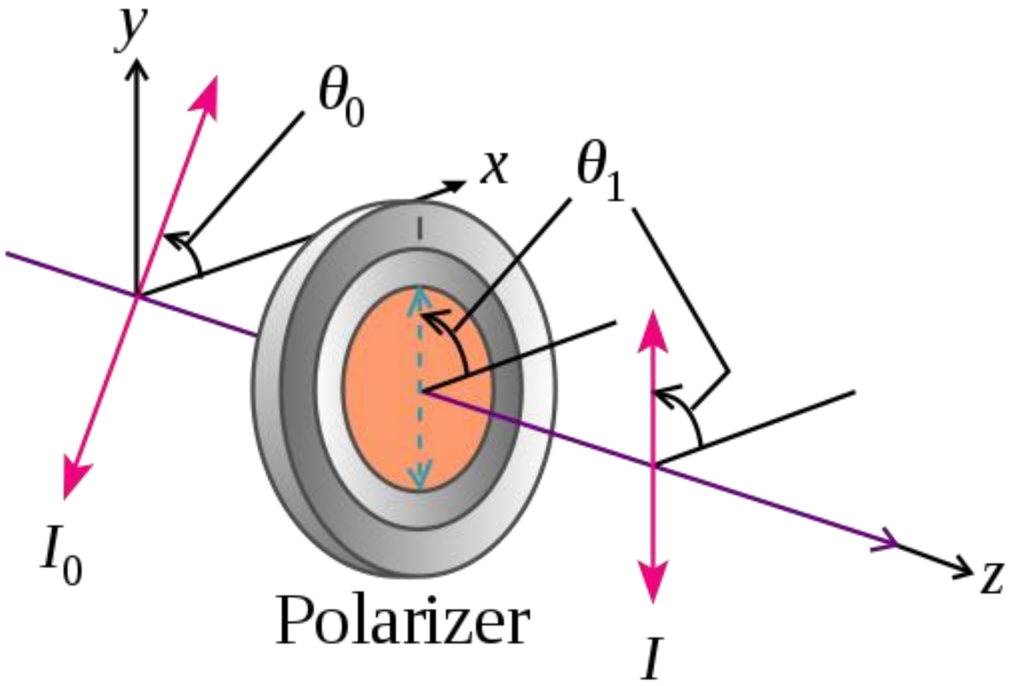


$$|E_{trans}| = |E_0| cos\theta$$

$$\rightarrow\ I = I_0 \cos^2 \theta_i \rightarrow I_{avg} = \frac{1}{2}$$

Polarization – Malus' Law X-rays





generally

$$I = I_0 \cos^2 \theta_i \rightarrow I_{avg} = \frac{1}{2}$$

Including relativistic effects (x-rays)

$$I = \frac{I_0 f}{\mathrm{f}_0}\left[1 + \frac{\lambda(f_0 - f)}{2c}\right]\cos^2 \theta_i$$