Fundamental Concepts – Constants and how they were determined.

# Modern Physics

PHILLIP WU 2020.9.14

- Some topics we will cover

  - Units and constants
  - Atomic mass of proton, neutron and electron as well as the Avogadro’s number
  - Millikan’s Oil drop
  - Interferometry
  - Relativity
  - Classical Wave Phenomena
  - Blackbody radiation (more in depth as a check)

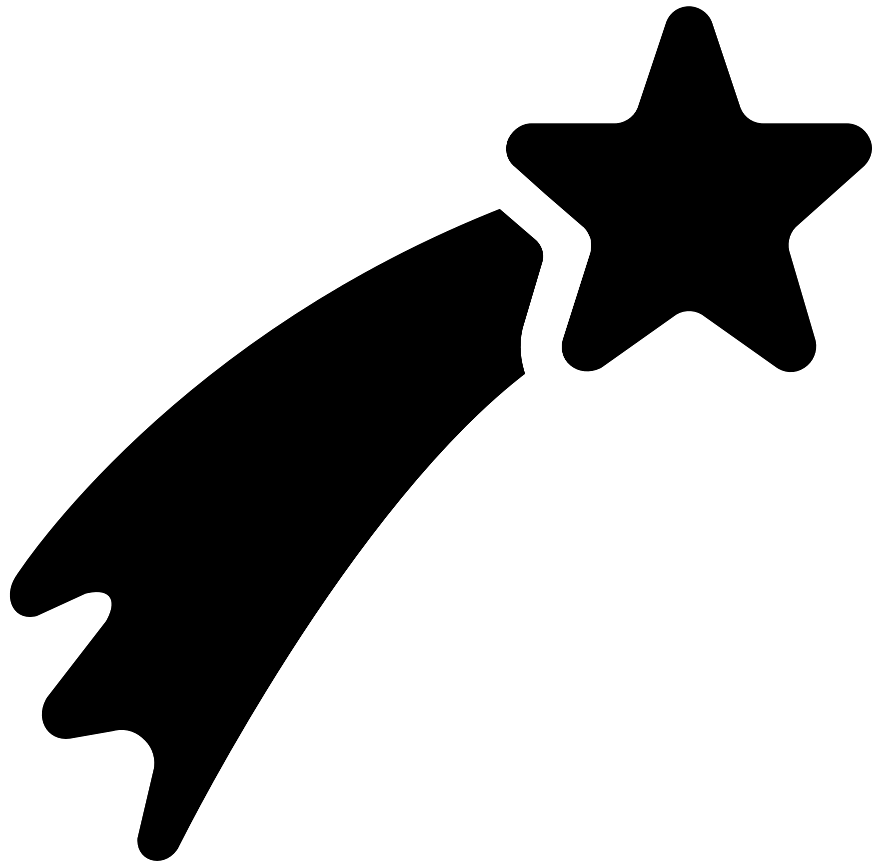
The fundamental constants of the universe tell us that specific numbers quantify and underpin the very fabric of our existence.



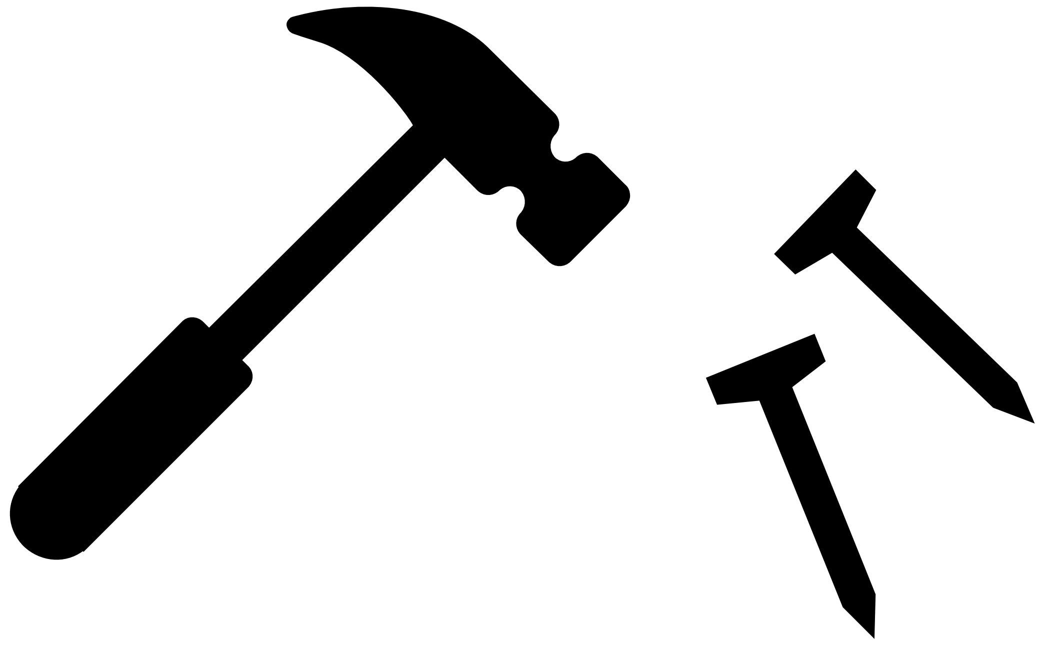
From Handbook of Chemistry & Physics

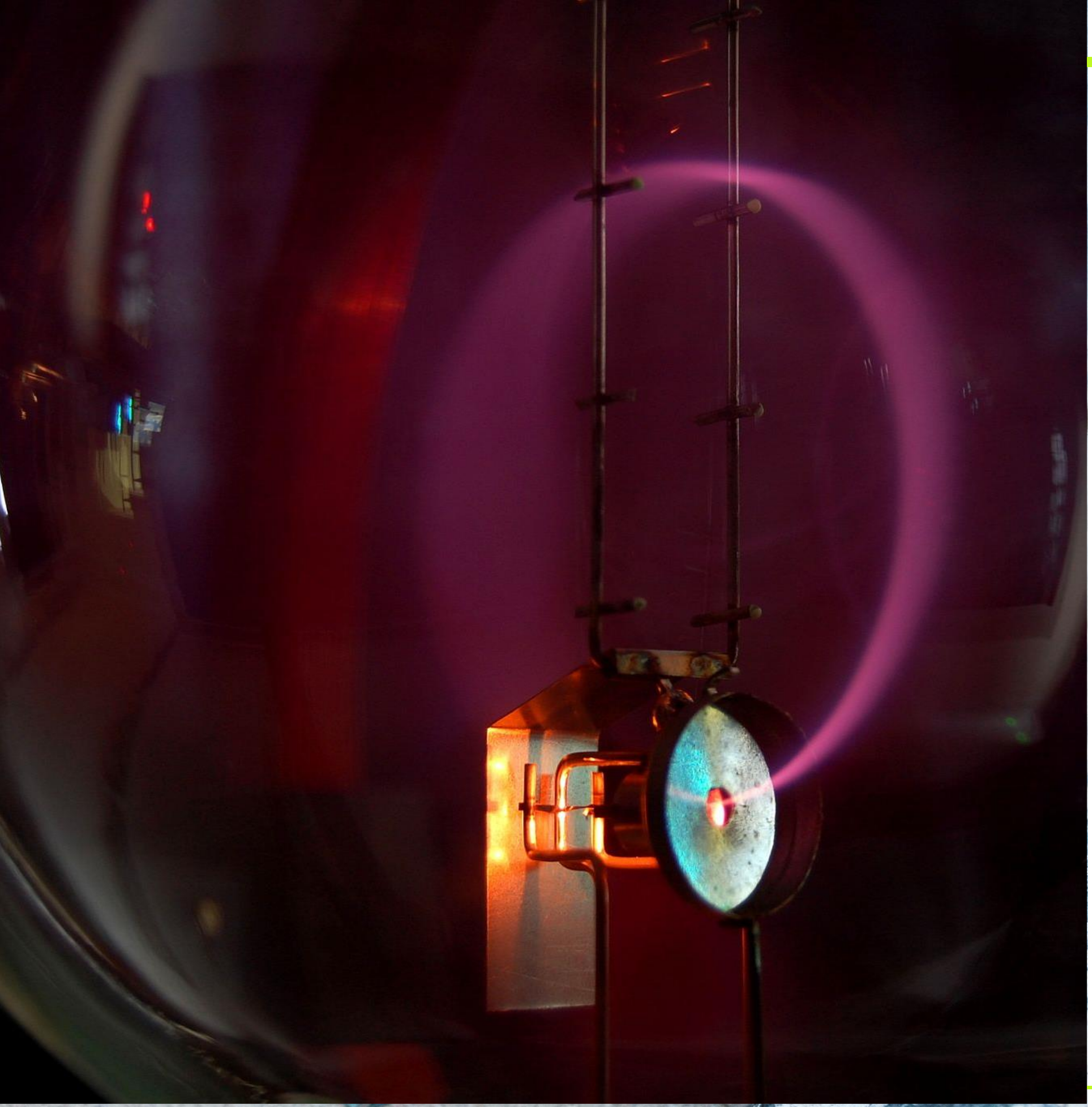
## Electron, e-

- Electron mass $m_e$ = 9.109 383 56 (11) x $10^{-31}$ kg with relative standard uncertainty $1.2 \times 10^{-8}$
- Energy equivalent $m_e c^2$ = 8.187 105 65 (10) x$10^{-14}$ J with relative standard uncertainty $1.2 \times 10^{-8}$ [also expressed in MeV]
- Electron-neutron mass ratio $m_e/m_n$ = 5.438 673 4428 (27) x10^-4 with relative standard uncertainty 4.9x10^-10
- Electron-proton mass ratio $m_e/m_p$ = 5.446 170 213 52 (52) x10^-4 with relative standard uncertainty 9.5x10^-11
- Electron molar mass $N_A m_e$ = M(e) = 5.485 799 090 70 (16) x10^-7 kg/mol with relative standard uncertainty 2.9x10^-11
- Compton wavelength $h/m_e c = L_{C,n}$ = 2.426 310 2367 (11) x10^-12 m with relative standard uncertainty 4.5x10^-10
- Electron magnetic moment $\mu_e$ = -928.476 4620 (57) x10^-26 J/T with relative standard uncertainty 6.2x10^-9
- Electron gyromagnetic ratio $2|\mu_e|/\hbar = \gamma_n/2\pi$ = 28 024.951 64 (17) MHz/T with relative standard uncertainty 6.2x10^-9




玻璃隕石 Taktite
成份：SiO2 發現地：中國雲南
說明：此類隕石分布在中國南方及中南半島一帶，雖然說是隕石，但其成因卻是當大隕石撞擊地球時所產生的高熱及高爆將原來附近地表上的二氧化矽熔化，噴向大氣層後再掉下來的產物，故表面有因為急速燃燒所形成氧化現象的隕石特徵。
感謝 陳明正先生捐贈
2007年6月17日下午17:30分
於桃園永安漁港拾得
鎳鐵隕石 Meteorite
產地：外太空
比重：7.165～7.64
發現地：中國廣西 Ganxi, China





By Marcin Białek - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=5257178

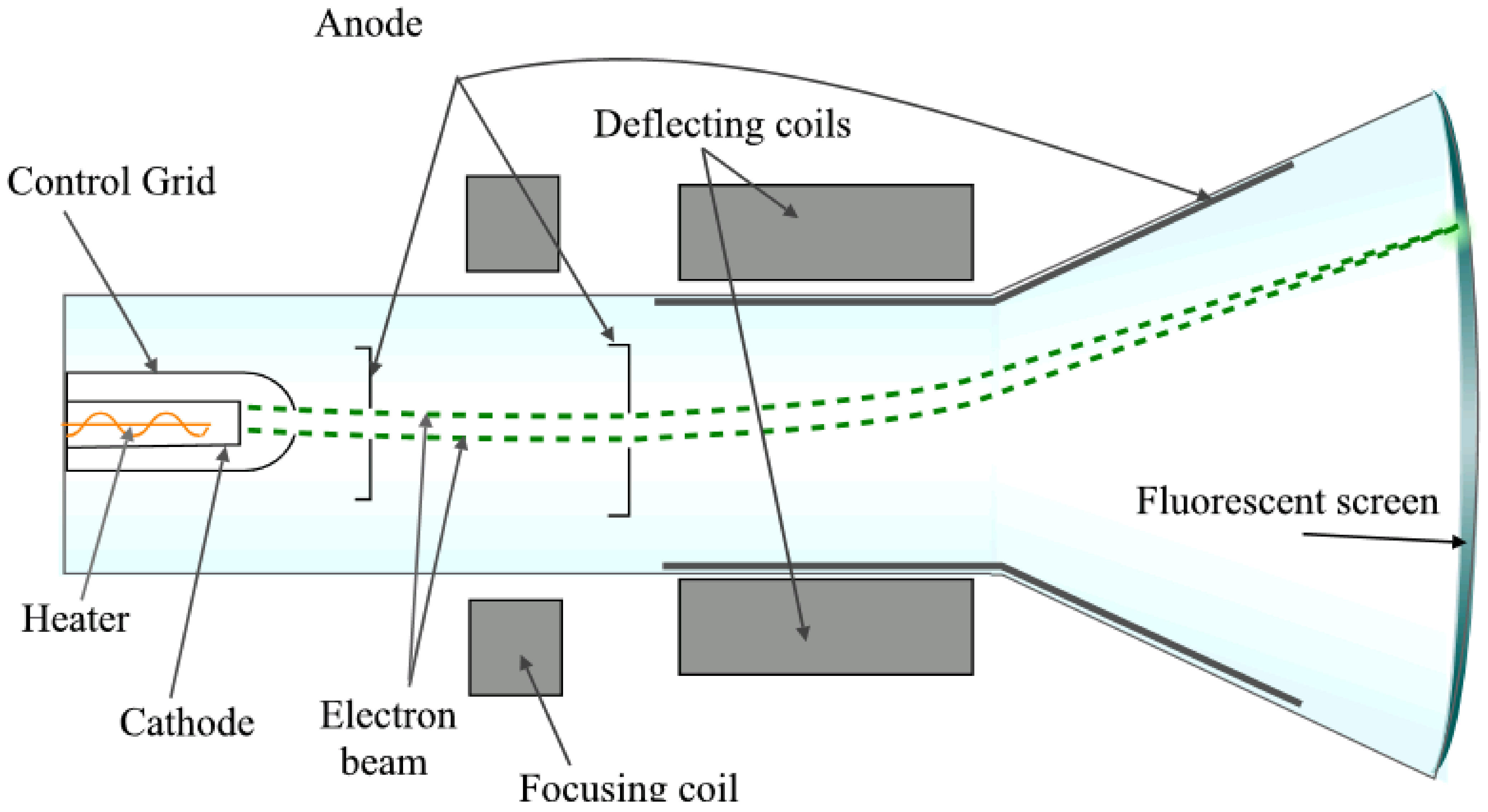
# The electron mass and charge

- Mass to charge ratio in magnetic cathode ray tubes 1890, Arthur Schuster and J J Thomson
- Robert Millikan’s oil drop experiment
- Spectroscopic techniques from the Rydberg constant and fine structure constant

Image on left: Beam of cathode rays (electrons) moving in a circle in a magnetic field (cyclotron motion). The electrons are produced by an electron gun at bottom, consisting of a hot cathode, a metal plate heated by a filament so it emits electrons, and a metal anode *(right)* at a high voltage with a hole which accelerates the electrons into a beam. Cathode rays are normally invisible, but enough air has been left in the tube so that the air molecules glow pink from fluorescence when struck by the fast-moving electrons.

# Cathode Ray Tubes

- Magnetic fields to bend electron path.
- Measurement gives the mass to charge ratio.




Caption
By Theresa Knott - en:Image:Cathode ray Tube.PNG, CC BY-SA 3.0,
https://commons.wikimedia.org/w/index.php?curid=100143

Caption
https://commons.wikimedia.org/wiki/User:Sergei_Frolov

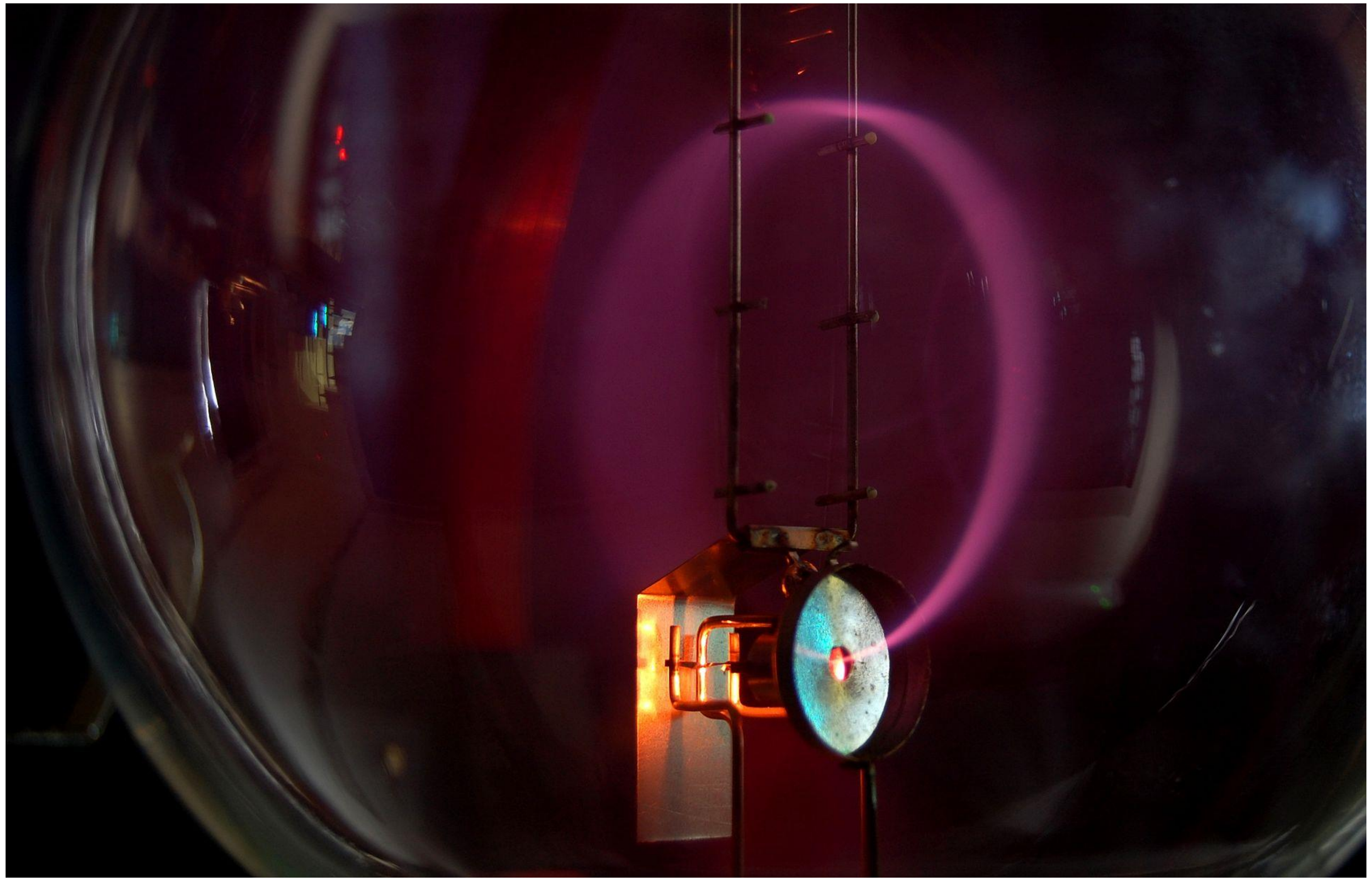
$$F = q(E + vxB)$$

$$F = ma = m\frac{dv}{dt}$$

$$(\frac{m}{q})a = E + vxB$$

# Cathode Ray Tubes

- Magnetic fields to bend electron path.
- Measurement gives the mass to charge ratio.



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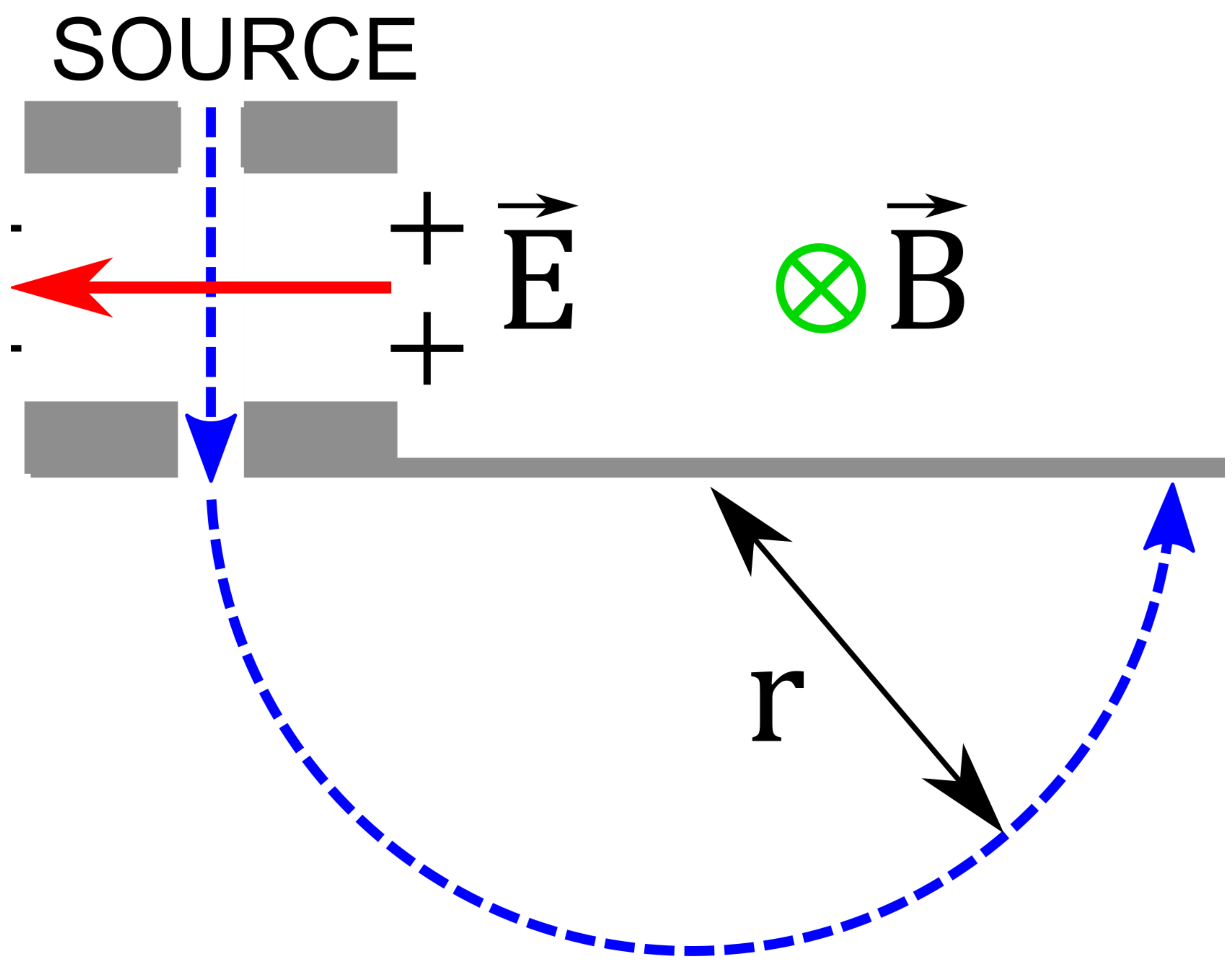



Caption

$$F = q(E + vxB)$$

$$F = ma = m\frac{v^2}{r}$$

$$\left(\frac{m}{q}\right)a = E + vxB$$

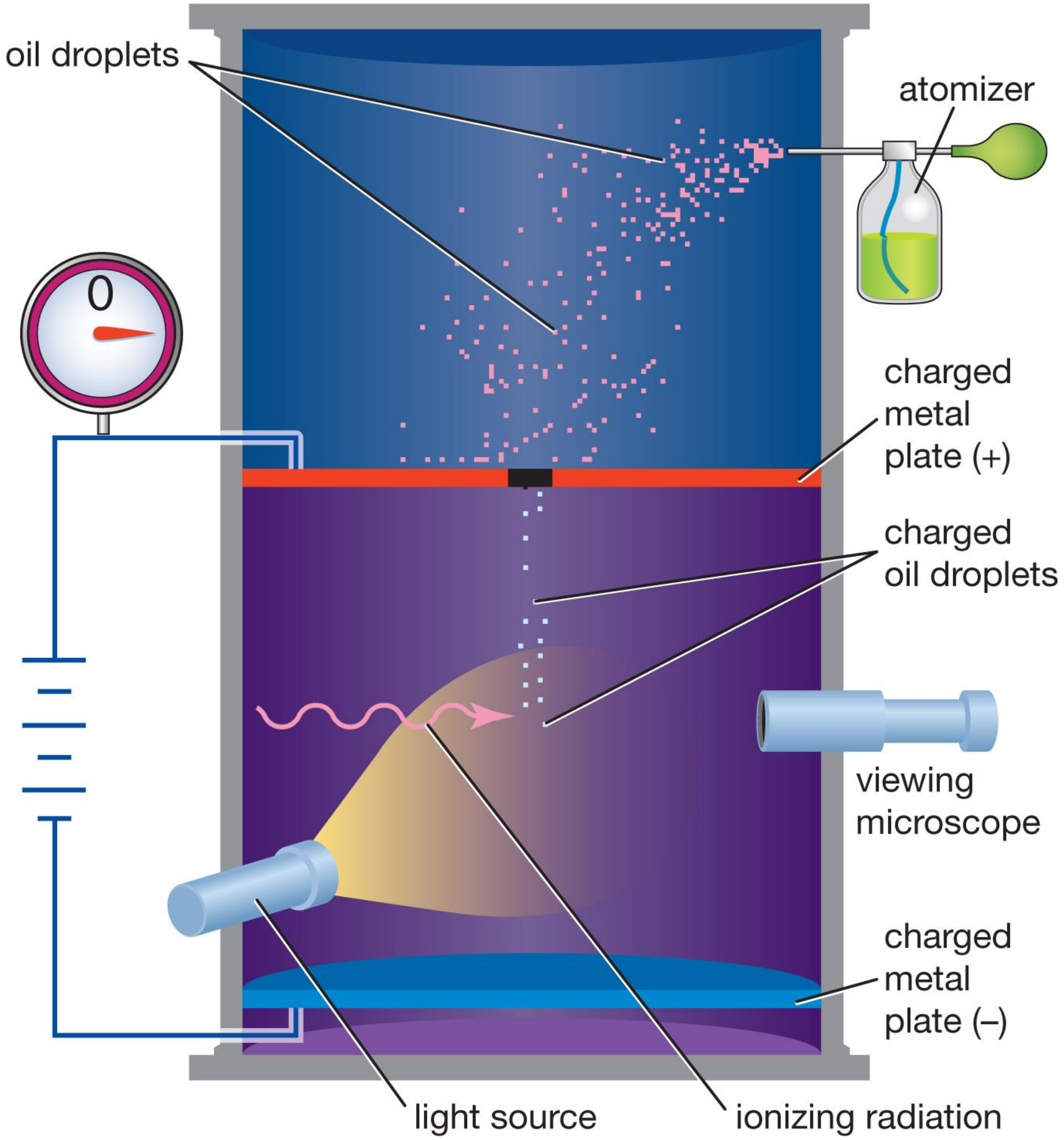
$$\frac{q}{m} = \frac{E}{B^2 r}$$

$-e/m_e = -1.75882001076(53)\times10^{11}\ C{\cdot}kg^{-1}$

# Millikan’s Oil Drop Experiment

Millikan

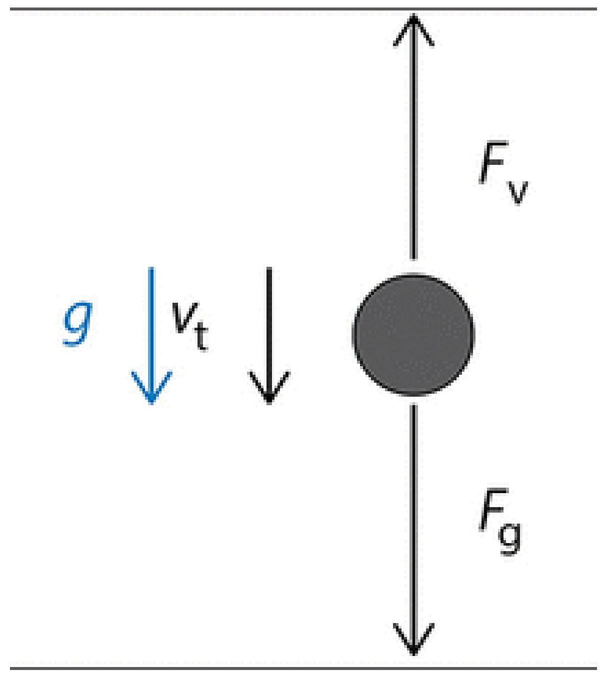
- Millikan’s experiment was designed to determine the electron charge. The weight of a single droplet is $w = mg = \frac{4\pi}{3} r^3 (\rho - \rho_{air}) g$
- Oil droplets sprayed into vertical vacuum chamber and allowed to drop downwards from gravity. Electrodes to generate electric field. $E = V/d$
- The force experienced is $F = qE = qV/d$
- At sufficient field strength, the electric field balances the weight of a single droplet dropping from gravity (including the viscous drag forces). $F_d = 6\pi r\eta v_1$
- That is, $qE - w = 6\pi r\eta v_2 \rightarrow q = (6\pi r\eta v_2 + \frac{4\pi}{3} r^3 (\rho - \rho_{air} g))/E$



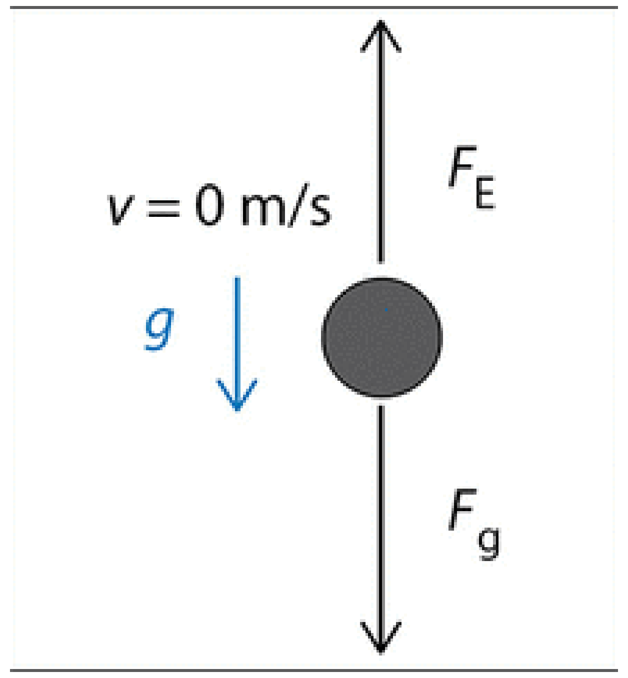




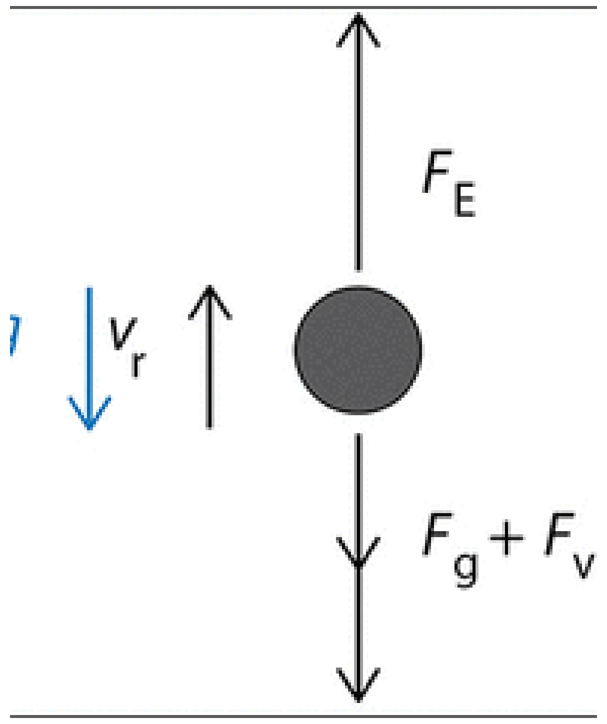

https://www.britannica.com/science/Millikan-oil-drop-experiment




In free fall



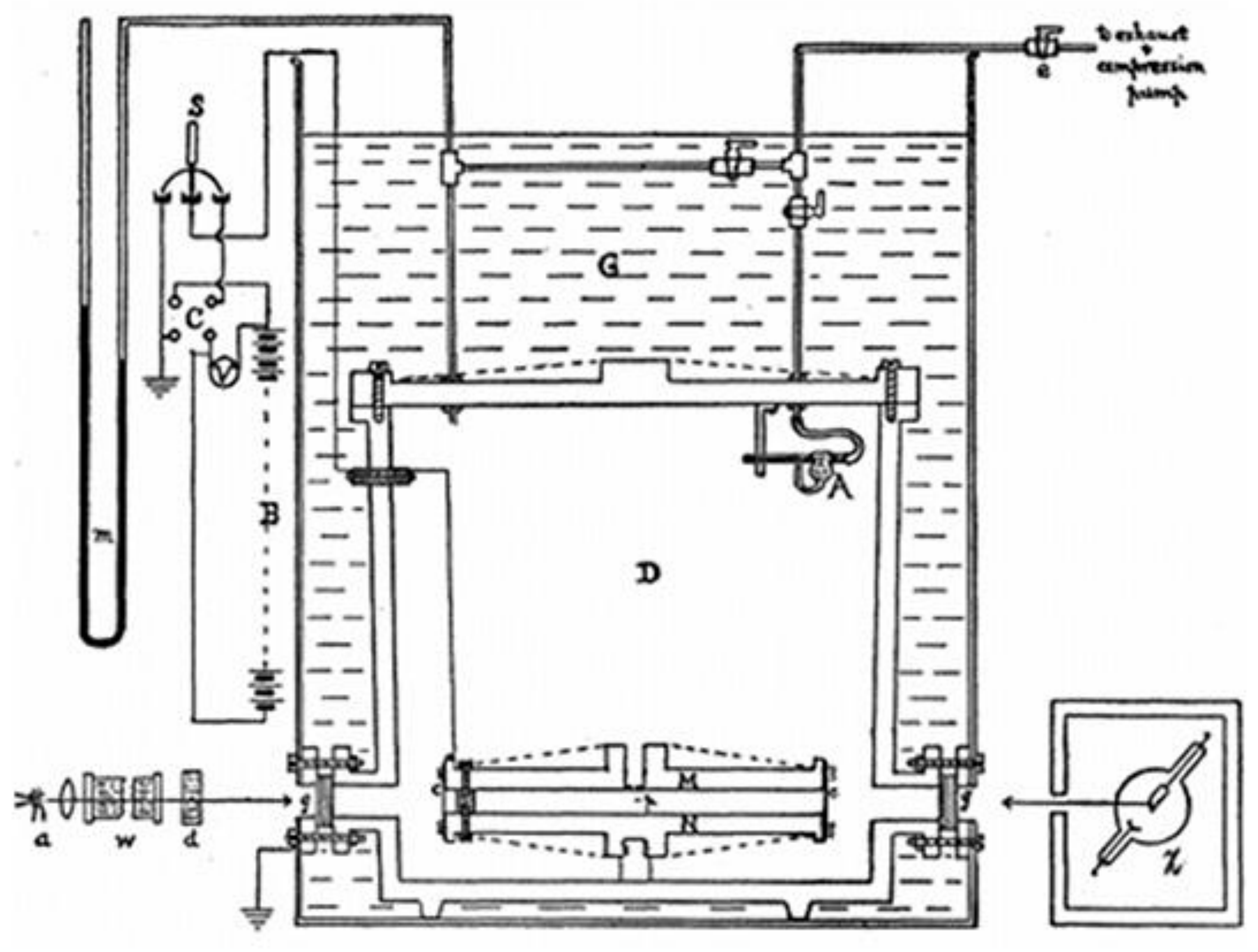

Suspended in electric field




Rising in electric field

$$q = (6\pi r \eta v_2 + \frac{4\pi}{3} r^3 (\rho - \rho_{air} g))/E \rightarrow r^2 = \frac{9\eta v_t}{2\pi g(\rho - \rho_{air})}$$

Millikan measured the terminal velocity of the droplet, and from that determined the mass, and then the electron charge




*KEY*

| | | | |
|---|---|---|---|
| **D** | Brass, pressure sealed vessel up to 15 atmospheres | **d** | Chloride cell |
| **m** | Mercury manometer acting as a standard barometer at atmospheric pressure | **p** | The oil droplet |
| **M,N** | Condenser plates | **G** | Temperature bath of gas-engine oil (40 litres) |
| ***g*** | Glass windows (3 in total, only 2 shown in diagram) | $\mathcal{X}$ | Röntgen ray emitter |
| **w** | Water cell | **A** | Atomizer |
| | | **e** | Small tube delivering dry, dust-free air to the atomizer |
| | | **c** | Ebonite strip encircling the condenser plates |

# Millikan's Oil Drop Experiment

- Some parameters:

| | | |
|---|---|---|
| Air viscosity | $\eta$ | 1.83 x10$^{-5}$ Newton seconds meter$^{-2}$ |
| Oil density | $\rho$ | 874 kg m$^{-3}$ at 20 degrees Celsius |
| Air density | $\rho_{air}$ | 1.30 kg m$^{-3}$ at 20 degrees Celsius |
| Plate Spacing | D | 6.00 x 10$^{-3}$ m |
| Electronic Charge | E | 1.60 x 10$^{-19}$ Coulomb |
| Electron Mass | M | • 9.109 383 56 (11) x 10^-31 kg |

# Spectroscopic Techniques

- This requires understanding of Planck's constant, h, and the basic quantum description of an atom (such as the Bohr model). Bohr found that the wavelength of emitted or absorbed light in vacuum has a relation that goes as $\frac{1}{\lambda} = R_M(\frac{1}{n_1^2} - \frac{1}{n_2^2})$ . This function allows determination of wavelength of light from for example the hydrogen atom during transitions. Importantly, as we will see, the Energy, wavelength and frequency of light are connected,

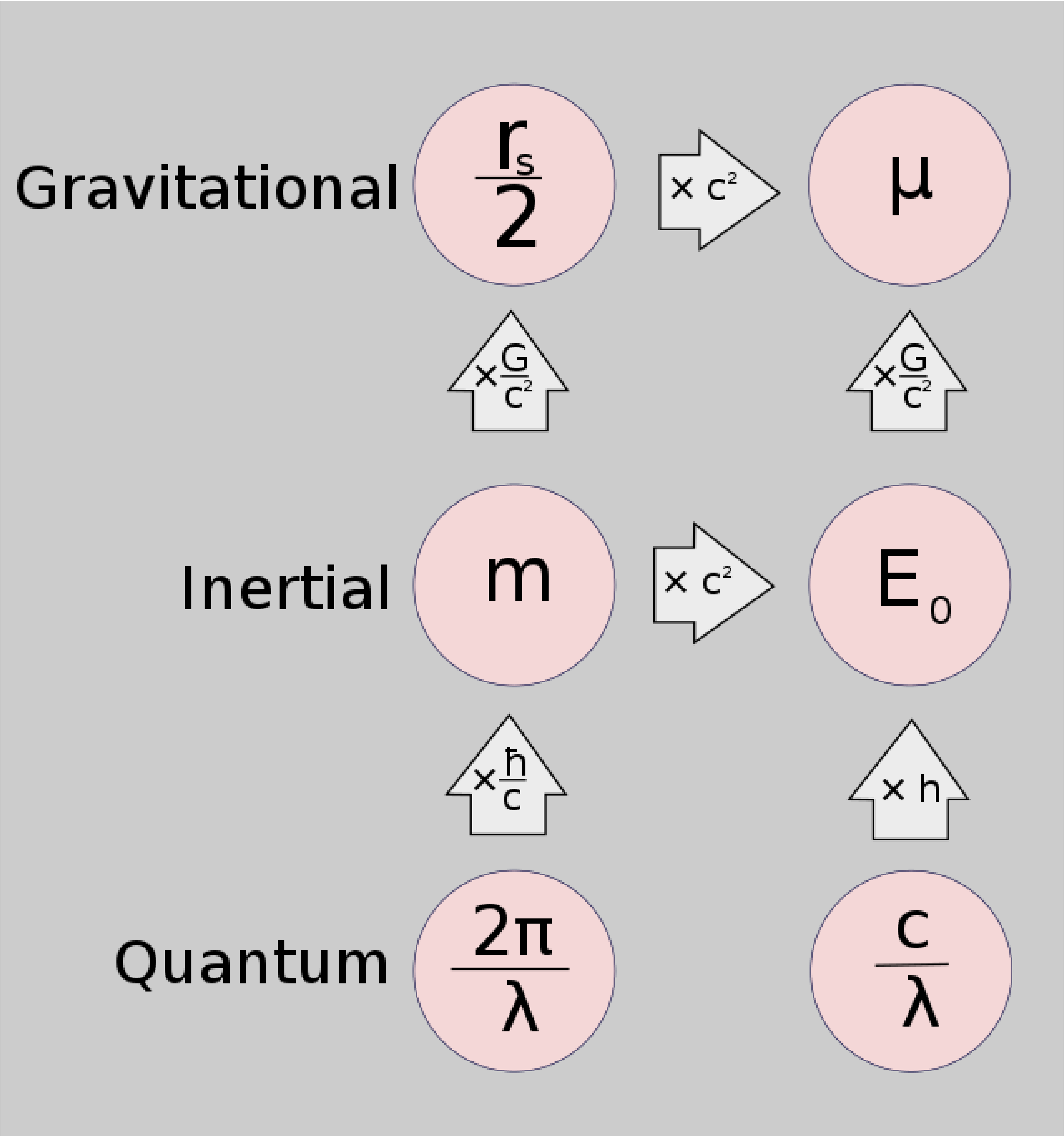
$$E = h\nu = \frac{hc}{\lambda}.$$

- The Rydberg constant is defined as $R_M = hcR_{inf} \rightarrow R_{inf} = \frac{m_e c\alpha^2}{2h} = \frac{m_e e^4}{8\epsilon_0^2 h^3 c}$/ (21) 160 731.568 973 10=m.
- For hydrogen, the simplest atom, the Rydberg constant is $R_H = R_{inf}\frac{m_p}{m_e+m_p} = 1.09678x10^-7/m$

- Spectroscopic techniques form the basis of many of the physics probes in the modern era. We will discuss optics concepts next week.

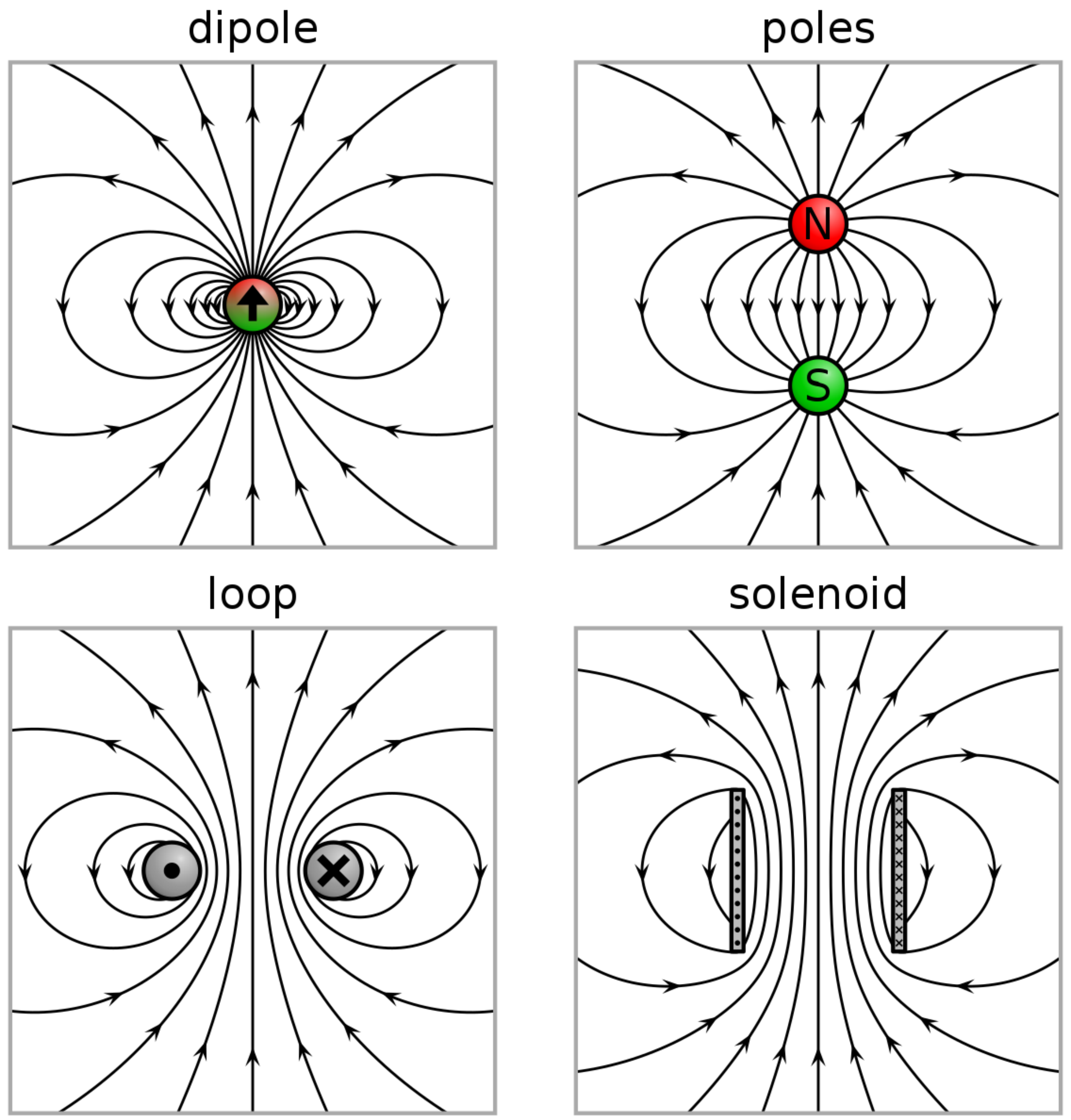
# Compton Wavelength

- The Compton wavelength of a particle, after Arthur Compton, is the wavelength of the photon whose energy is the same as the mass of that particle. $\lambda = \frac{h}{mc}$ with $f = \frac{mc^2}{h}$. For the electron this value is 2.426 310 2367 (11) x10^-12 m with relative standard uncertainty 4.5x10^-10.
- The reduced (inverse) Compton wavelength is a natural representation of mass on the quantum scale. (take the lambda expression and divide by 2π).




Parameters and concepts defining quantum, inertial, gravitational concepts

# The Electron Magnetic Moment and Gyromagnetic Ratio




- Arises from considering the effect of magnetic field on the energetic state of the electron or particles – also called the Zeeman effect (considerations to first order). Experimentally we can quantify the moment as

$\mu = -\frac{\partial Energy}{\partial B}$ and the magnetization as $\chi = \frac{M}{H} = \frac{\mu_0 M}{B}$.

- The description of an electron moving for example in a circular orbit is its angular momentum L, which interacts with the external magnetic field to give an energy

$\Delta E(L) = \frac{-e}{2m_0} L \cdot B$. The electron has also an intrinsic "spin," akin to thinking of a top spinning on its axis.

$\Delta E(S) = \frac{-e}{m_0} S \cdot B$.

- The electron magnetic moment is then the interacting term, $\mu = \frac{-e}{2m_0} L$ and the gyromagnetic ratio is $-e/2m * m_0$

Image here of moment pp. 222 Beiser

- We will discuss in further detail the spin of electrons with the Stern-Gerlach experiment (Oct. 12), and magnetism of materials starting (Oct. 19).



https://commons.wikimedia.org/wiki/File:VFPt_dipoles_magnetic.svg#/media/File:VFPt_dipoles_magnetic.svg

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# Periodic table of the elements

- Alkali metals
- Alkaline-earth metals
- Transition metals
- Other metals
- Other nonmetals
- Halogens
- Noble gases
- Rare-earth elements (21, 39, 57–71) and lanthanoid elements (57–71 only)
- Actinoid elements

| period \ group | 1* | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 H | | | | | | | | | | | | | | | | | 2 He |
| 2 | 3 Li | 4 Be | | | | | | | | | | | 5 B | 6 C | 7 N | 8 O | 9 F | 10 Ne |
| 3 | 11 Na | 12 Mg | | | | | | | | | | | 13 Al | 14 Si | 15 P | 16 S | 17 Cl | 18 Ar |
| 4 | 19 K | 20 Ca | 21 Sc | 22 Ti | 23 V | 24 Cr | 25 Mn | 26 Fe | 27 Co | 28 Ni | 29 Cu | 30 Zn | 31 Ga | 32 Ge | 33 As | 34 Se | 35 Br | 36 Kr |
| 5 | 37 Rb | 38 Sr | 39 Y | 40 Zr | 41 Nb | 42 Mo | 43 Tc | 44 Ru | 45 Rh | 46 Pd | 47 Ag | 48 Cd | 49 In | 50 Sn | 51 Sb | 52 Te | 53 I | 54 Xe |
| 6 | 55 Cs | 56 Ba | 57 La | 72 Hf | 73 Ta | 74 W | 75 Re | 76 Os | 77 Ir | 78 Pt | 79 Au | 80 Hg | 81 Tl | 82 Pb | 83 Bi | 84 Po | 85 At | 86 Rn |
| 7 | 87 Fr | 88 Ra | 89 Ac | 104 Rf | 105 Db | 106 Sg | 107 Bh | 108 Hs | 109 Mt | 110 Ds | 111 Rg | 112 Cn | 113 Nh | 114 Fl | 115 Mc | 116 Lv | 117 Ts | 118 Og |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| lanthanoid series 6 | 58 Ce | 59 Pr | 60 Nd | 61 Pm | 62 Sm | 63 Eu | 64 Gd | 65 Tb | 66 Dy | 67 Ho | 68 Er | 69 Tm | 70 Yb | 71 Lu |
| actinoid series 7 | 90 Th | 91 Pa | 92 U | 93 Np | 94 Pu | 95 Am | 96 Cm | 97 Bk | 98 Cf | 99 Es | 100 Fm | 101 Md | 102 No | 103 Lr |

*Numbering system adopted by the International Union of Pure and Applied Chemistry (IUPAC).

## Proton, p

- Proton mass $m_p$ = 1.672 621 898 (21) x $10^{-27}$ kg with relative standard uncertainty $1.2x10^{-8}$
- Energy equivalent $m_p c^2$ = 1.503 277 593 (18)x$10^{-10}$ J with relative standard uncertainty $1.2x10^{-8}$
- Proton-electron mass ratio $m_p / m_e$ = 1836.152 673 89 (17) with relative standard uncertainty $9.5x10^{-11}$
- Proton-neutron mass ratio $m_p / m_n$ = 0.998 623 478 44 (51) with relative standard uncertainty $5.1x10^{-10}$
- Proton charge to mass quotient $e/ m_p$ = 9.578 833 226 (59) x$10^7$ C/kg with relative standard uncertainty $6.2x10^{-9}$
- Proton molar mass $N_A m_p$ = M(p) = 1.007 276 466 879 (91) x$10^{-3}$ kg/mol with relative standard uncertainty $9.0x10^{-11}$
- Proton Compton wavelength $h/ m_p c = LC_p$ = 1.321 409 853 96 (61) x$10^{-15}$ m with relative standard uncertainty $4.6x10^{-10}$
- Proton rms charge radius $r_p$ = 0.8751 (61) x$10^{-15}$ m with relative standard uncertainty $7.0x10^{-3}$
- Proton magnetic moment $\mu_p$ = 1.410 606 7873 (97) x$10^{-26}$ J/T with relative standard uncertainty $6.9x10^{-9}$
- Proton gyromagnetic ratio $2 \mu_p / \hbar = \gamma_p /2\pi$ = 42.577 478 92 (29) MHz/T with relative standard uncertainty $6.9x10^{-9}$

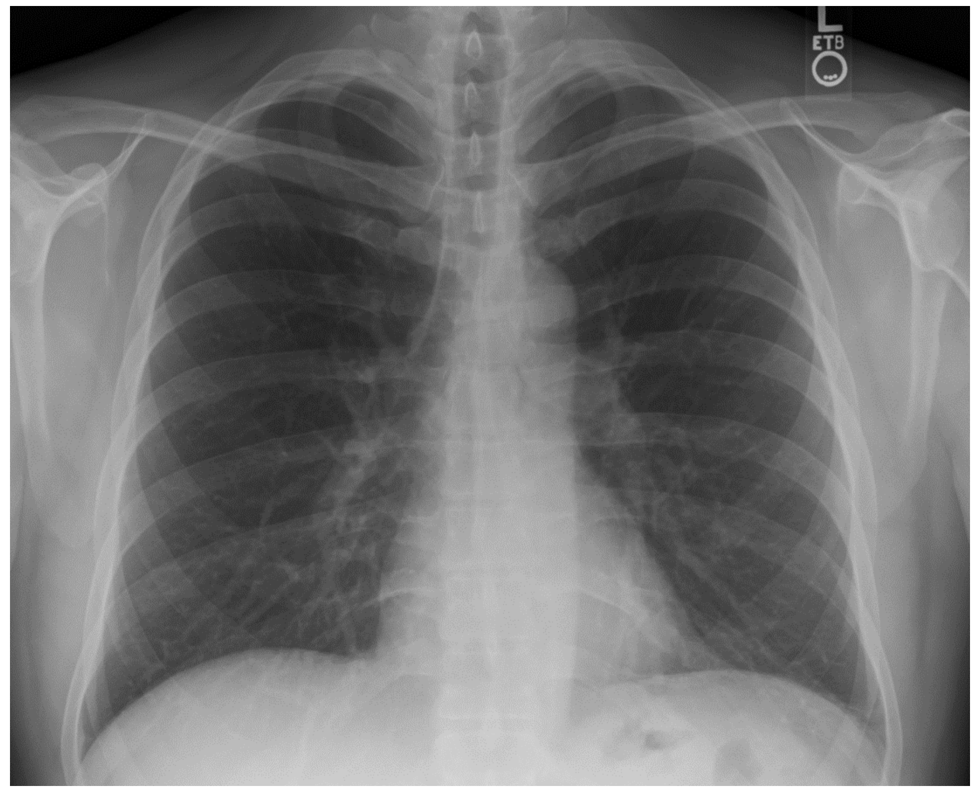
## Neutron, n

- Neutron mass $m_n$ = 1.674 927 471 (21) x 10^-27 kg with relative standard uncertainty 1.2x10^-8
- Energy equivalent $m_n$ c^2 = 1.505 349 739 (19)x10^-10 J with relative standard uncertainty 1.2x10^-8
- Neutron-electron mass ratio $m_n$ / $m_e$ = 1838.683 661 58 (90) with relative standard uncertainty 4.9x10^-10
- Neutron-proton mass ratio $m_n$ / $m_p$ = 1.001 378 418 98 (51) with relative standard uncertainty 5.1x10^-10
- Neutron molar mass $N_A$ $m_n$ = M(n) = 1.008 664 91588 (49) x10^-3 kg/mol with relative standard uncertainty 4.9x10^-10
- Neutron Compton wavelength h/ $m_n$ c = $LC,_n$ = 1.319 590 904 81 (88) x10^-15 m with relative standard uncertainty 6.7x10^-10
- Neutron magnetic moment $\mu_n$ = -0.996 236 50 (23) x10^-26 J/T with relative standard uncertainty 2.4x10^-7
- Neutron gyromagnetic ratio 2| $\mu_n$ |/hbar = $\gamma_n$ /2π = 29.164 6933  MHz/T with relative standard uncertainty 2.4x10^-7

# From the Nucleus and Upwards

- From the nucleus, proton (and neutron) spins and moment, arises techniques such as nuclear magnetic resonance (NMR) and magnetic resonance imaging (MRI), Medical Physics topics we will discuss Dec. 28.

- The nucleus is also the basis of our nuclear physics understanding, which we will discuss in early January.

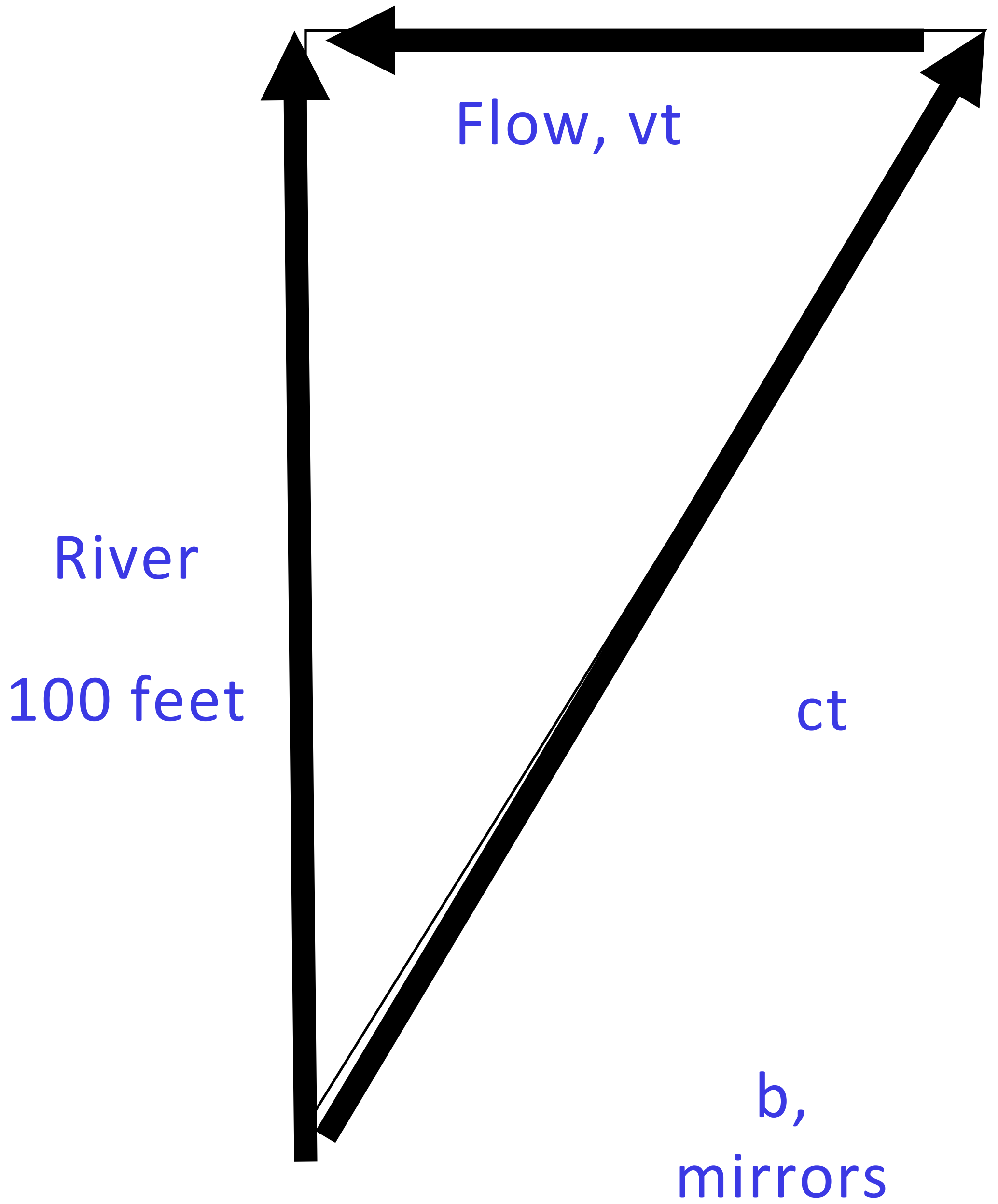


## Alpha Particle, alpha

- Alpha particle mass $m_\alpha$ = 6.644 657 230 (82) x 10^-27 kg with relative standard uncertainty 1.2x10^-8
- Energy equivalent $m_\alpha$ c^2 = 5.971 920 097 (73) x10^-10 J with relative standard uncertainty 1.2x10^-8
- Alpha particle-electron mass ratio $m_\alpha / m_e$ = 7294.299 541 36 (24) with relative standard uncertainty 4.9x10^-11
- Alpha particle-proton mass ratio $m_\alpha / m_p$ = 3.972 599 689 07 (36) with relative standard uncertainty 9.2x10^-11
- Alpha particle molar mass $N_A\, m_\alpha$ = M(α) = 4.001 506 179 127 (63) x10^-3 kg/mol with relative standard uncertainty 1.6x10^-11

# Gravity and Relativity

- From the energy equivalent, the relation of mass and energy is defined. Already this well known concept tells us that something is special about the relationship between energy, mass and light.
- Let’s see why light is such a special thing.
- Consider a swimmer trying to cross perpendicularly the river bank .
- Michelson had this thought experiment: *Suppose we have a river of width w (say, 100 feet), and two swimmers who both swim at the same speed v feet per second (say, 5 feet per second). The river is flowing at a steady rate, say 3 feet per second. The swimmers race in the following way: they both start at the same point on one bank. One swims directly across the river to the closest point on the opposite bank, then turns around and swims back. The other stays on one side of the river, swimming upstream a distance (measured along the bank) exactly equal to the width of the river, then swims back to the start. Who wins?*

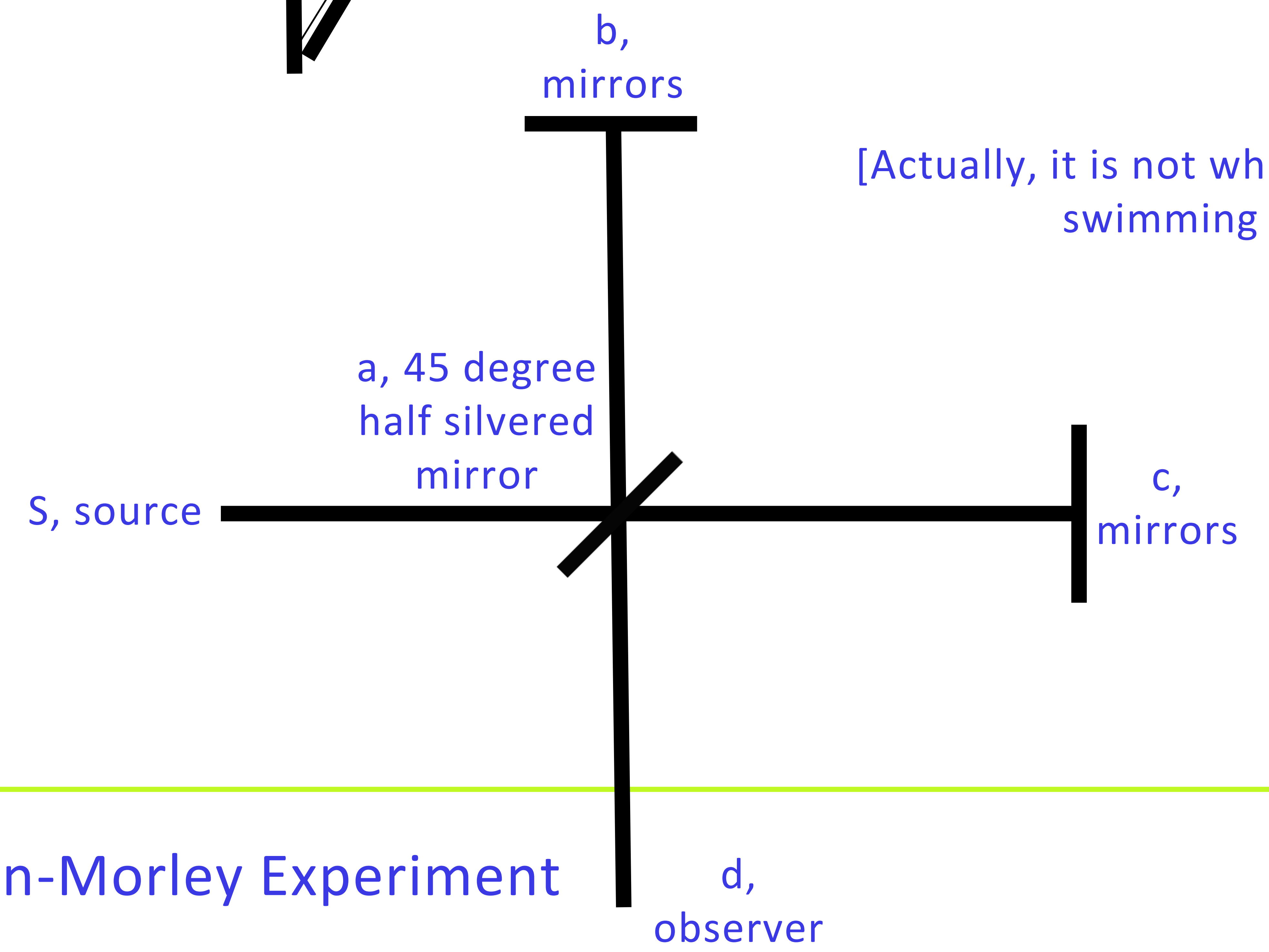



In time t, the swimmer crossing the bank moves diagonally ct relative to water, and has been carried vt distance by water flow.

While the other swimmer swims along one bank edge for also 100 feet, then back to the start.

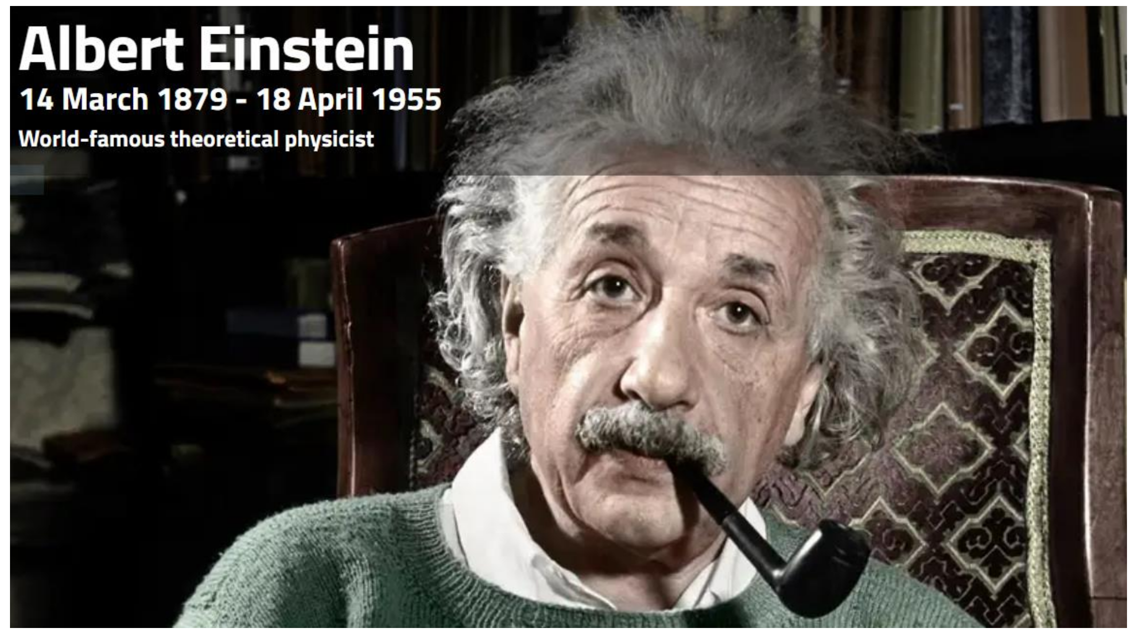
Turns out, swimmer crossing the bank will take shorter time to return to the start (if swimming the same distance).

[Actually, it is not who wins but who is swimming faster]



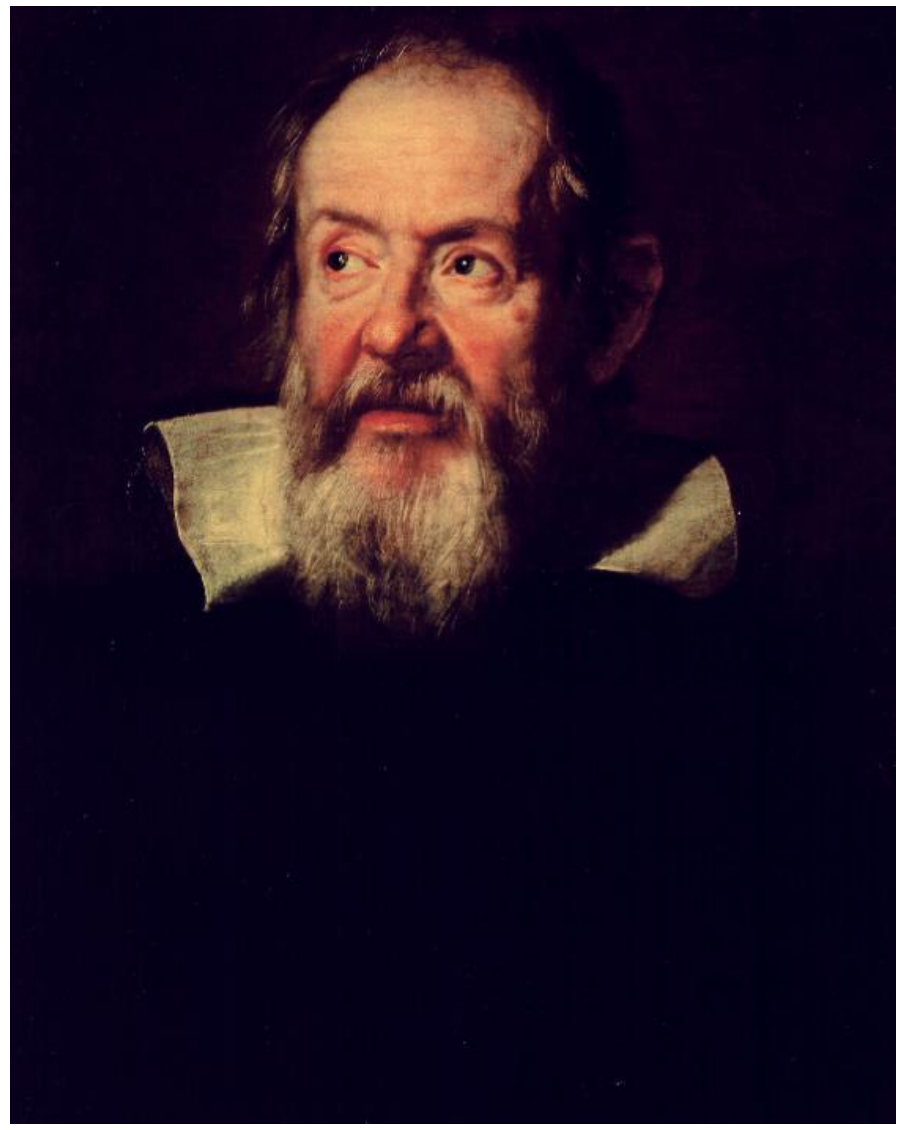

The Michelson-Morley Experiment

# Michelson-Morley Experiment and the Postulates of Relativity




- Mathematically:
- $t_{across-stream} + t_{back} = \frac{2w}{\sqrt{(c^2-v^2)}} = \frac{2w}{c}\left(1+\frac{1}{2}\left(\frac{v^2}{c^2}\right)\right) \sim \frac{2w}{c}\left(1+\frac{v^2}{2c^2}\right), \frac{v}{c} \ll 1,$

$$\sqrt{1-x} \sim 1-\frac{1}{2}x, \frac{1}{1-x} \sim 1+x$$

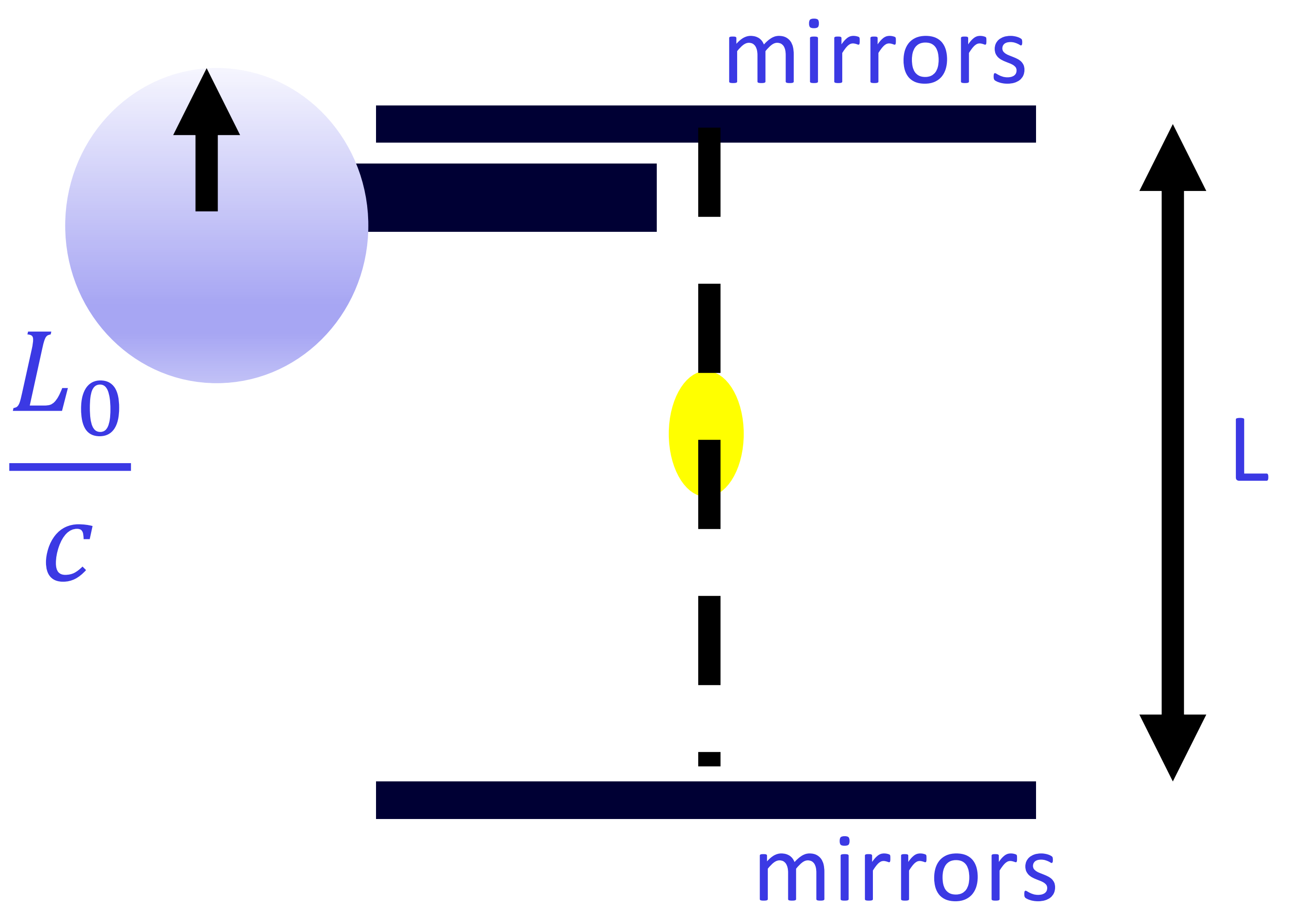


- Michelson repeated the experiment in numerous ways, modifying apparatus, measuring on top of a high mountain in California, and observed no difference in the time light took to traverse the two different paths.
- The conclusion: that light does not travel via a medium such as sound in air or waves in water.
- And! Light has a speed that is a constant.
- From this, several important concepts in relativity were stated: “The Laws of Physics are the same in all Inertial Frames” (Galileo’s principle of relativity restated by Einstein) and “The speed of light in free space has the same value in all inertial frames of reference”.
- That value for light is c = 3 * $10^8$ m/s.

# Time Dilation and Fitzgerald Contraction

1. "The Laws of Physics are the same in all Inertial Frames" (Galileo's principle of relativity restated by Einstein)
2. "The speed of light in free space has the same value in all inertial frames of reference".

That value for light is c = 3 * $10^8$ m/s.



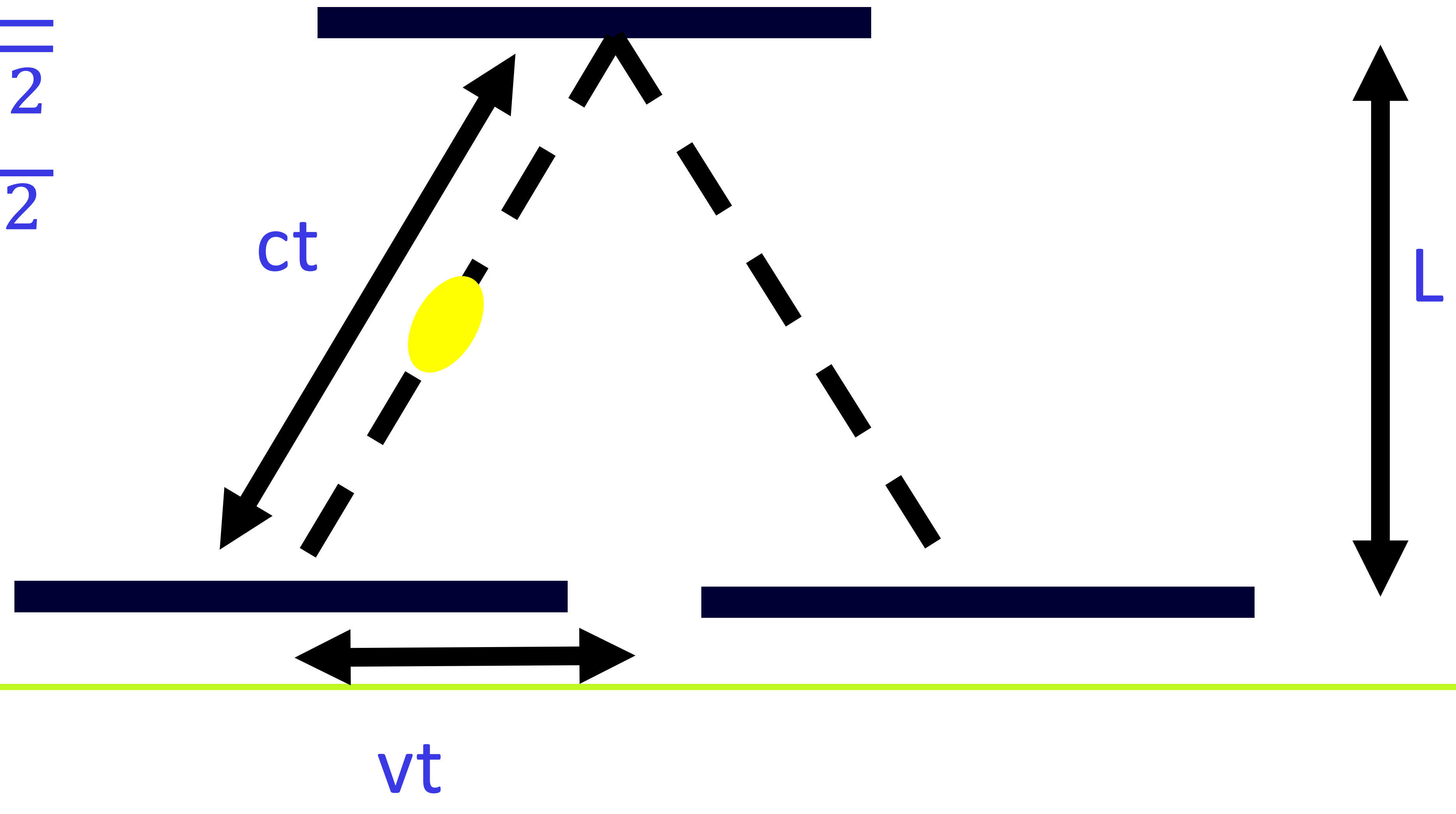

1. The path and time for the light traveling straight across or the light-pulse clock seen by observer on the ground. Each time light hits top mirror, clock moves on click-tik tok.

$$\frac{t_0}{2} = \frac{L_0}{c}$$

2. Suppose now the clock is moving at v,

$$\left(\frac{ct}{2}\right)^2 = L_0^2 + \left(\frac{vt}{2}\right)^2 \quad \text{or} \quad t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Time Dilation:
- http://galileoandeinstein.phys.virginia.edu/more_stuff/Applets/Lightclock/home.html



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2. "The speed of light in free space has the same value in all inertial frames of reference".

That value for light is c = 3 * $10^8$ m/s.

- Fitzgerald Contraction: even lengths are scaled by the factor, Y! Effectively, the faster moving object will seem shorter, in the direction of movement.
- This was verified with muon measurements, David H. Frisch and James A. Smith, "Measurement of the Relativistic Time Dilation Using Muons" American Journal of Physics, 31, 342 1963.
- The reality of Special Relativity is well illustrated by the GPS, which depends on very accurate clocks in moving satellites. The system yields seriously incorrect results if the relativistic clock slowing is ignored.

$$\Upsilon = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Example: the muon has speed 2.994 * $10^8$ m/s (0.998c) and lifetime 2.2 * $10^{-6}$ s

This gives $vt_0$ = 0.66 km

The lifetime of the muon observed by us, traveling not close to speed of light is t = gamma $t_0$ = 34.8 * $10^{-6}$ s

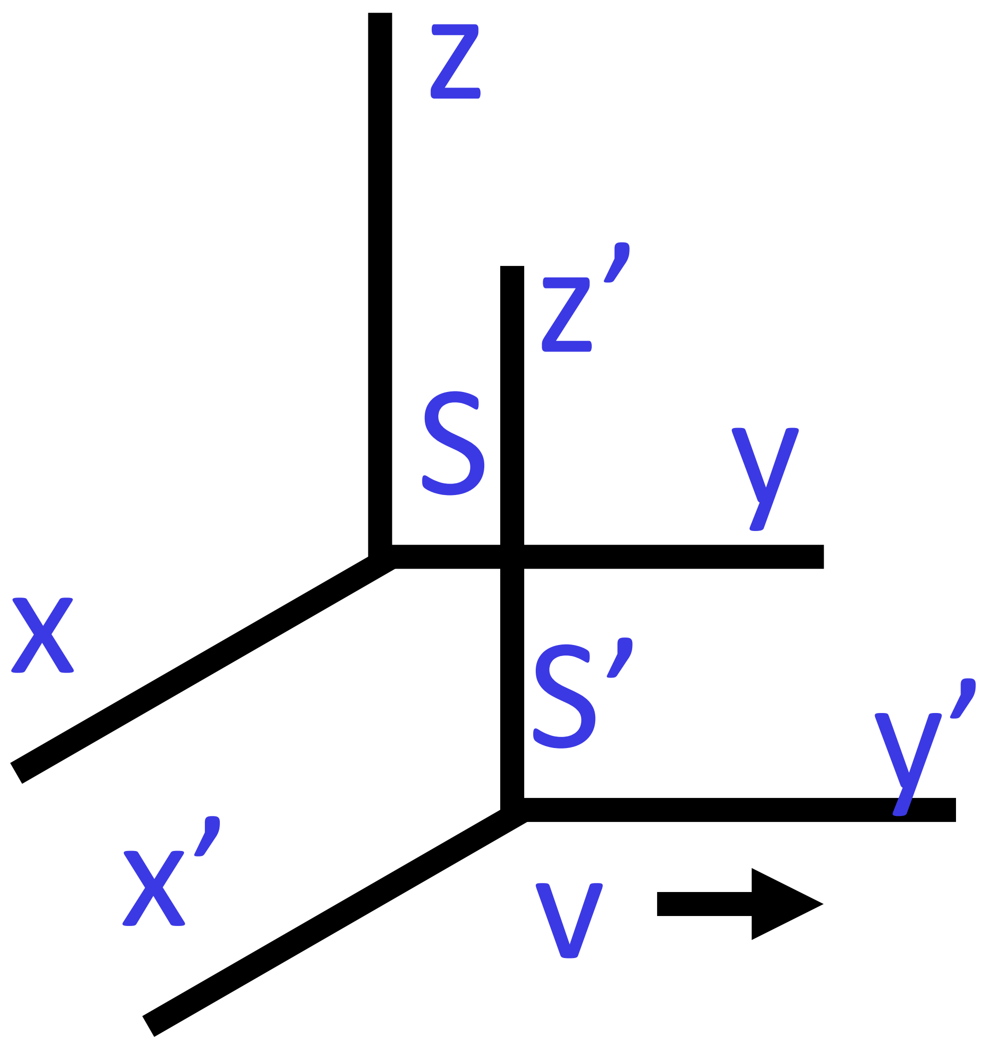
$$L = L_0 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

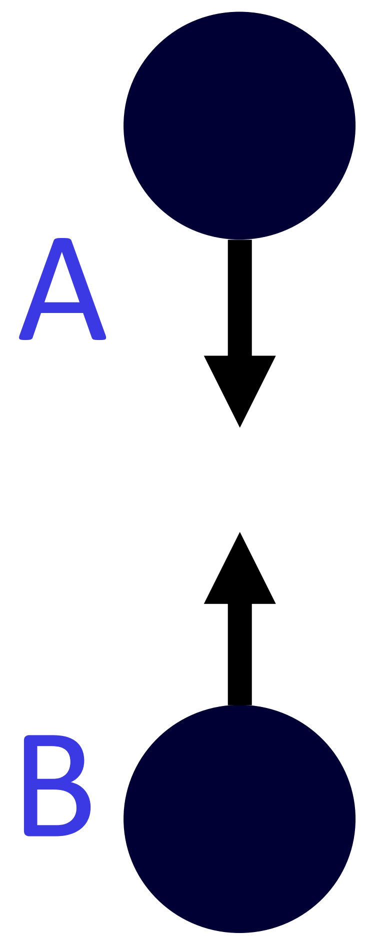
So we see and detect muons to be traveling vt = 10.4 km! (passing through the atmosphere)

The muon though, really only goes 0.66 km in its frame of reference, which is what we would observe too if traveling at 0.998c

# Mass and Energy

- The foundations of modern physics rests on the conservation laws of energy and momentum. Even considering that the velocity of light is fixed (though fast), energy conservation is found to still be true. The outcome of these is that an object's speed cannot keep increasing in proportion as more work is done on the object --- in other words, as the object's speed increases, so does its mass, so that the work done (from something) becomes kinetic energy of the object, while always keeping v less than or equal to c.




$$m_A\ V_A = m_B V_B \qquad V_A = \frac{Y}{T_0};\ V_B = \frac{Y}{T}$$

In the S' reference frame,

$$T = T_0 / \sqrt{1 - \frac{v^2}{c^2}}$$

So, $m_A = m_B \sqrt{1 - \frac{v^2}{c^2}}$

For the masses very small compared to the velocity, then the rest mass is on the left hand side ($m_0 = m_A$)

# Massless Particles

- Can a massless particle exist? (photons!) Turns out we can describe the electron and photon energies and momentums in the following way.

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}\ ; p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} \qquad E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

- An example: an electron with $m_0$ = 0.511 MeV/$c^2$ and a photon $m_0$ = 0 both have momenta of 2.000 MeV/$c^2$. The total energy is
- Electron $E = \sqrt{\{0.511\ MeV^2 + 2.000\ MeV^2\}} = 2.064\ MeV$
- Photon $E = pc = \left(2.000 \frac{MeV}{c}\right) c = 2.000\ MeV$

# Doppler Effect

- The increase in pitch of a sound when its source approaches us (or we approach the source) and the decrease in pitch when the source recedes from us (or we recede from the source). These changes in frequency constitute the Dopper Effect.

$$v = v_0 \left( \frac{1 + v/c}{1 - V/c} \right)$$

- Little “v” is the speed of observer and big “V” is the speed of the source.
- The Doppler Effect in sound varies depending on who or what is moving. Note that as sound waves exist in a material medium such as air or water, the medium itself is a frame of reference with respect to which motions of source and observer are measurable.
- Light, has no medium.

# Doppler Effect

- Observer moving perpendicular to a line between him/her and the light source. The proper time between ticks is $t_0 = 1/\nu_0$, so between ticks $t_0\,\gamma$ elapses in the reference frame of the observer. The frequency

$$\nu\ (transverse) = \frac{1}{t} = \frac{\sqrt{1-\frac{v^2}{c^2}}}{t_0} \rightarrow \nu = \nu_0\sqrt{1-\frac{v^2}{c^2}}$$

- The observed frequency is always lower than the source frequency.

- AN observer receding from the light source. Now the observer travels the distance vt away from the source between ticks, which means that the light wave from a given tick takes vt/c longer to reach him than the previous one.

$$T = t + \frac{vt}{c} = t_0\frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} = t_0\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

- The observed frequency is lower than the source frequency, regardless of the medium movement.

# Doppler Effect

- Observer approaching the light source. The observer here travels the distance vt toward the source between ticks so each light wave takes vt/c less time to arrive than the previous one. T=t-vt/c

$$\nu(longitudinal) = \nu_0 \sqrt{\frac{1+\frac{v}{c}}{1-\frac{V}{c}}}$$

# Doppler Effect – An example

- A driver is caught going through a red light. The driver claims to the judge that the color she/he saw was green (frequency = 5.6 * $10^{14}$ Hz) and not red (frequency$_0$ = 4.8 * $10^{14}$ Hz) because of the Doppler effect. The judge accepts this explanation and instead fines her/him for speeding at the rate of $1 for each km/h she/he exceed the speed limit of 80 km/h. What was the fine?

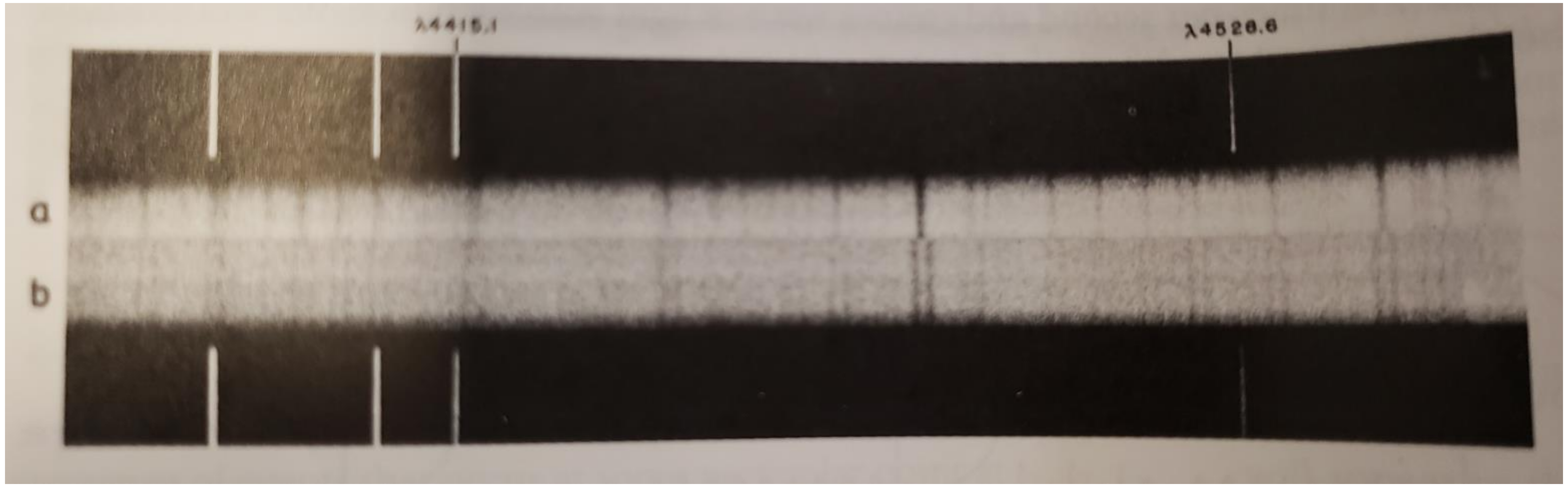
$$v = c\left(\frac{v^2 - v_0^2}{v^2 + v_0^2}\right) = 3 * 10^8\left(\frac{5.6^2 - 4.8^2}{5.6^2 + 4.8^2}\right) = 4.59 * \frac{10^7 m}{s} = 1.65 * 10^8 km/\mathrm{h}$$

- The fine is therefore $(1.65*$10^8$-80)=$164,999,920!!!

- This is the scale of astronomical experiments! The Doppler effect is actually very important in the study of astronomy

# Doppler Effect

- The observed frequency is higher than the source frequency. The convention is that v is + for source and observer approaching each other and – for source and observer receding from each other.




- The speeds of recession are observed to be proportional to distance, with the proportionality called Hubble’s Law.

# An example of the expanding universe

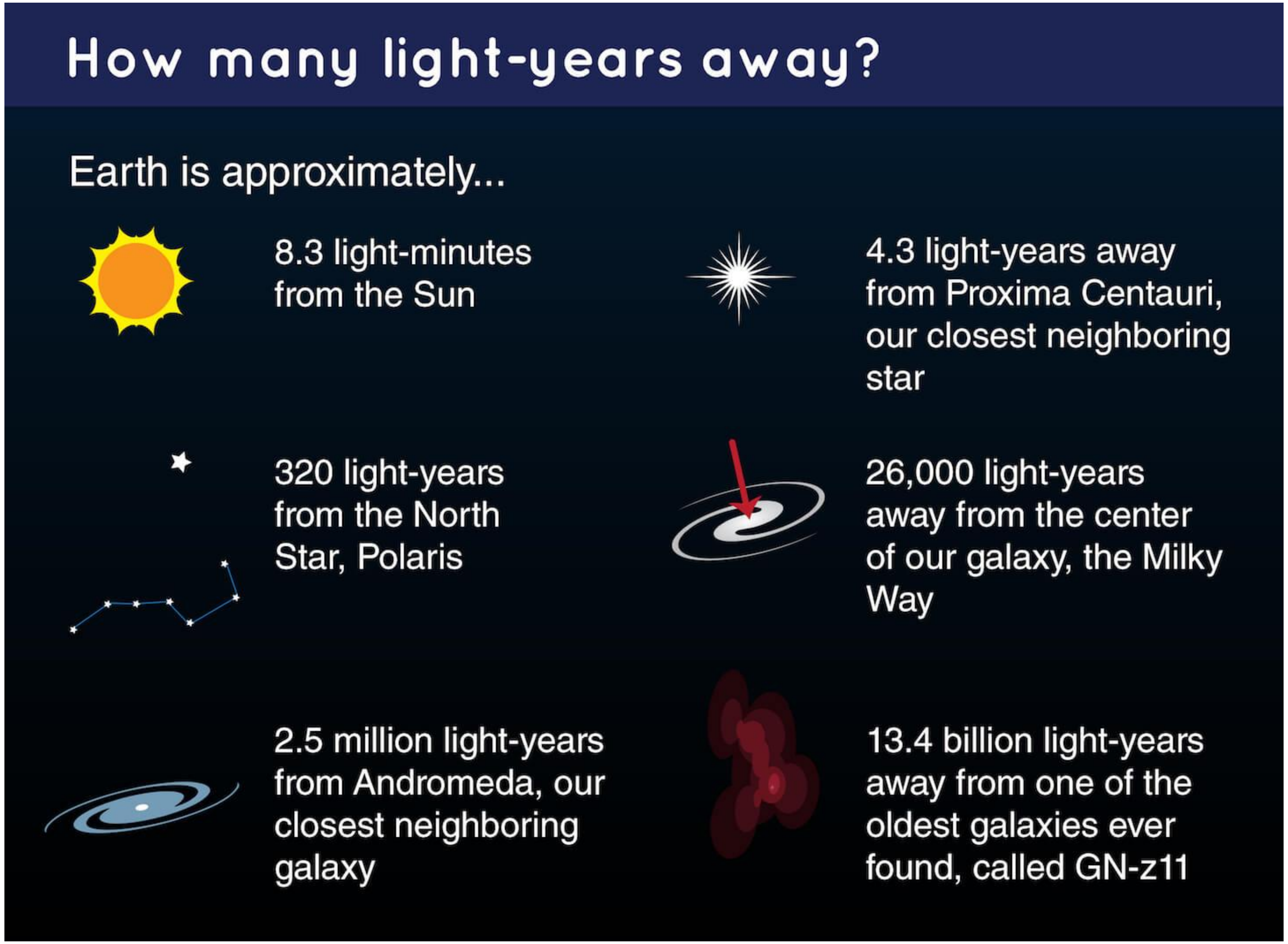
- A distant galaxy in the constellation Hydra is receding from the Earth at $6.12*10^7$ m/s. By how much is a green spectral line of wavelength 500 nm emitted by this galaxy shifted toward the red end of the spectrum?

$$\lambda = \frac{c}{\nu} \rightarrow \lambda = \lambda_0 \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

- Compared to the green spectral wavelength of 500 nm, the observed value is 615 nm, in the orange part of the spectrum. This shift of 115 nm tells us that the galaxy is nearly 3.6 billion light-years away.

1 light-year = 9460730472580800 metres (exactly)
≈ 9.461 petametres
≈ 9.461 trillion kilometres (5.879 trillion miles)
≈ 63241.077 astronomical units
≈ 0.306601 parsecs

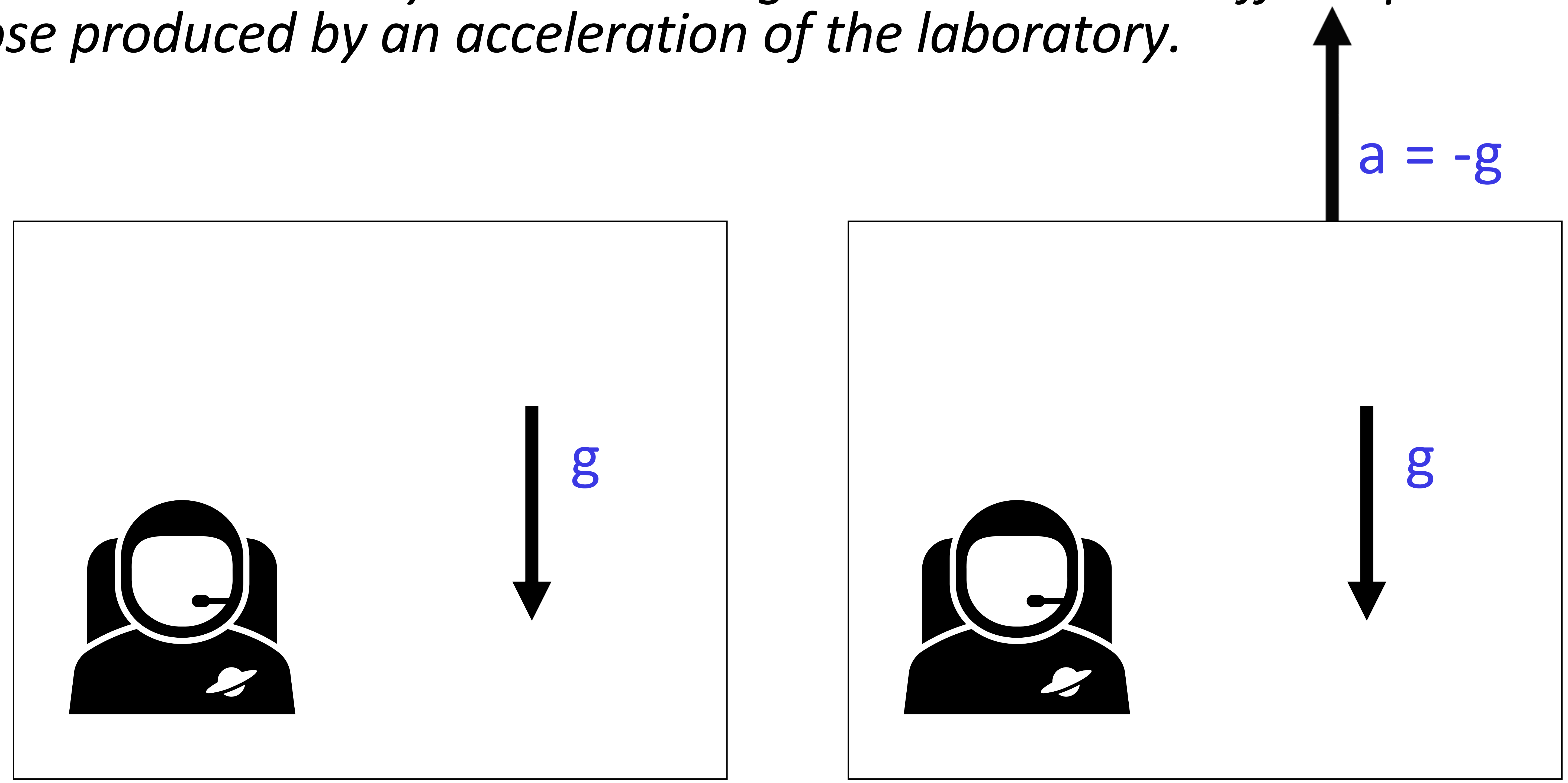
An image of distant galaxies captured by the NASA/ESA Hubble Space Telescope. Credit: ESA/Hubble & NASA, RELICS; Acknowledgment: D. Coe et al.




https://spaceplace.nasa.gov/light-year/en/

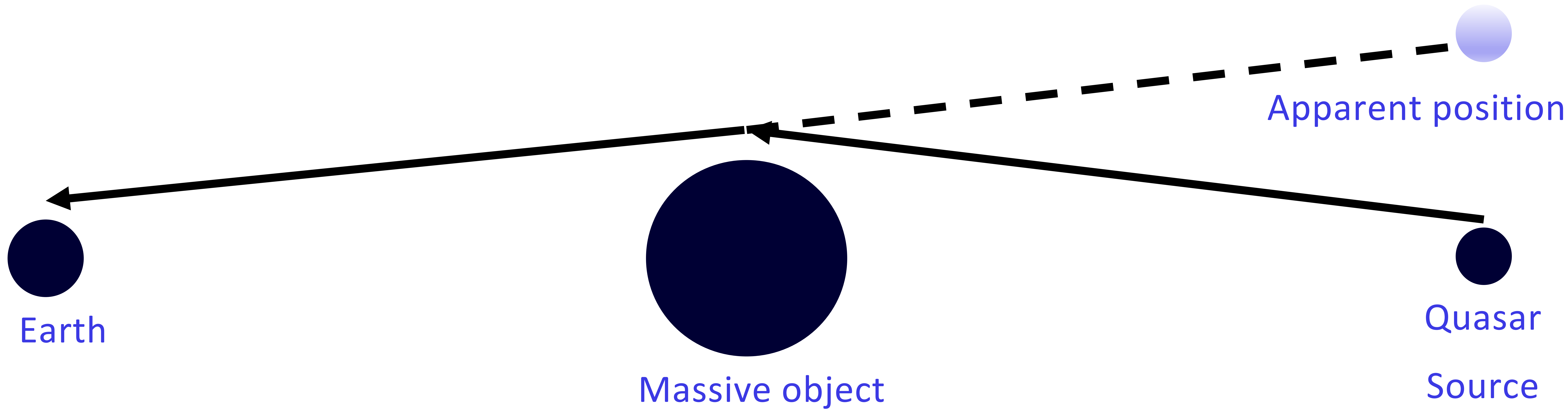
# General Relativity

- Special relativity is concerned with the inertial frames of reference, frames that are not accelerated.
- General relativity concludes that the force of gravity arises from a warping of spacetime around a body of matter (containing mass). The principal of equivalence is the observational conclusion (and postulated by Einstein): *An observer in a closed laboratory cannot distinguish between the effects produced by a gravitational field and those produced by an acceleration of the laboratory.*



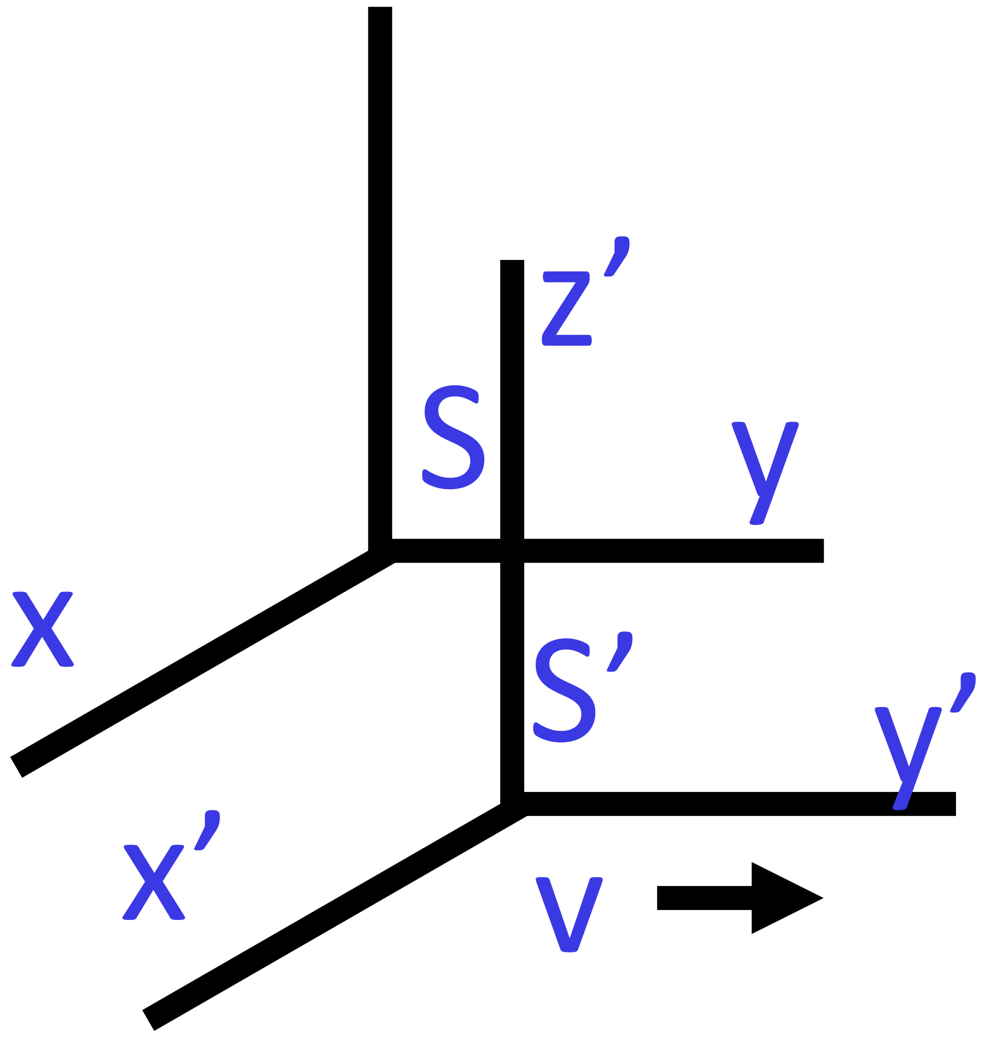
# General Relativity

- Other findings of General Relativity: Warping of the speed of light by massive gravitational objects and the confirmation of gravitational waves.

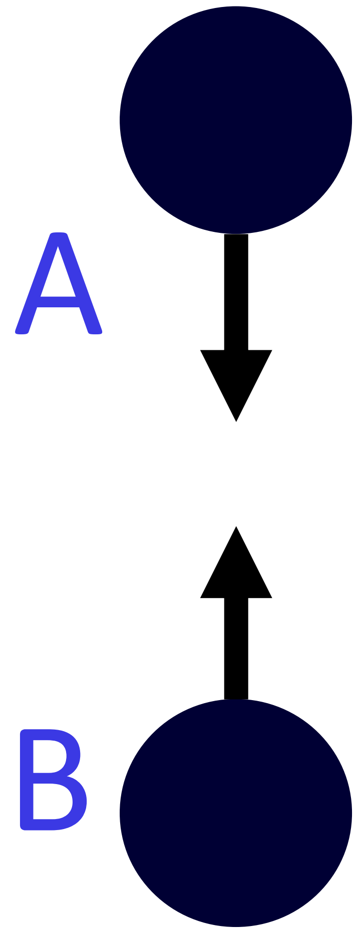


# Lorentz Transformation

- *Let's make sure to understand the transformations*
- *Before special relativity, transforming measurements from one inertial system to another seemed obvious. If clocks in both systems started when the origins S, S' coincide, then lengths vary by the speed times time of the moving system.*




$x' = x - vt \quad y' = y; z' = z$ and t'=t



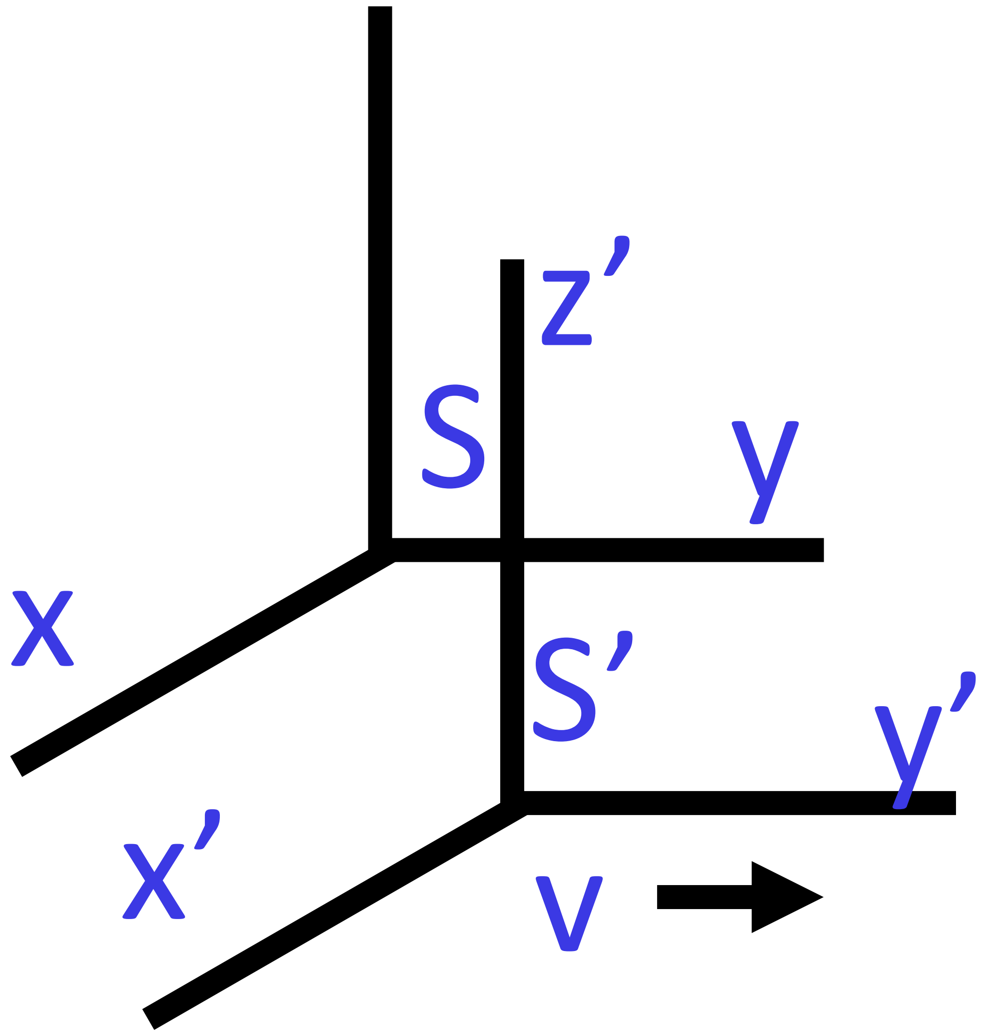

This is the Galilean transformation. In the S' reference frame, the velocities are simply the time derivatives.

$$v_x' = \frac{dx'}{dt'} = v_x - v; v_y' = \frac{dy'}{dt'} = v_y; v_z' = \frac{dz'}{dt'} = v_z$$

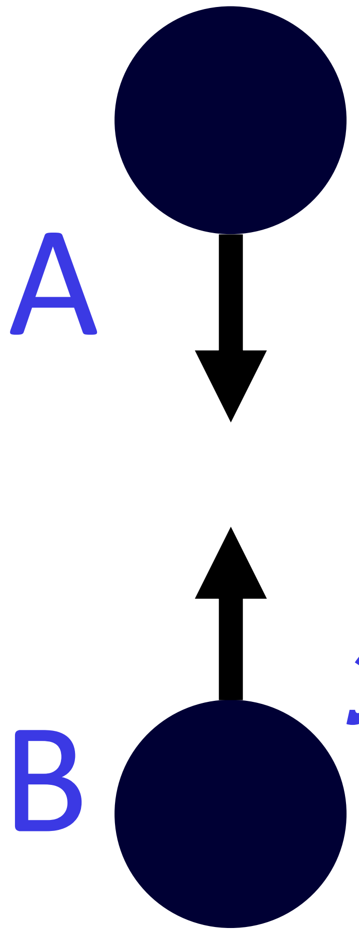
Using this transformation, the speed of light will work out to be c'=c-v

# Lorentz Transformation

- *Let's make sure to understand the transformations*
- *For a correct transformation, we make a guess based on the considerations that a single event in S must be also a single event in S', that we seek simple solutions and that when the speeds are close to "normal" we should get the Galilean before.*




$$x' = k(x - vt) \; or x = k(x' + vt') \qquad y' = y; z' = z$$



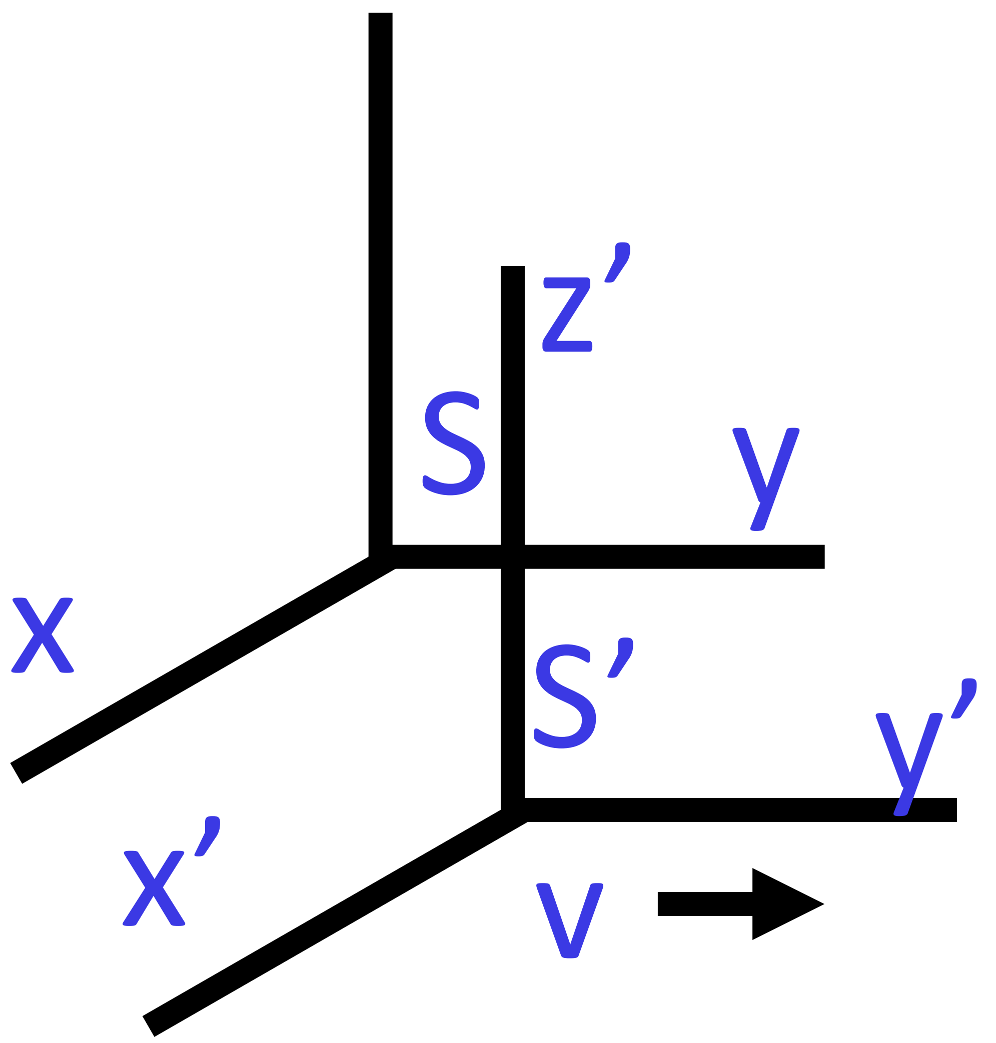

In time however, we have to consider

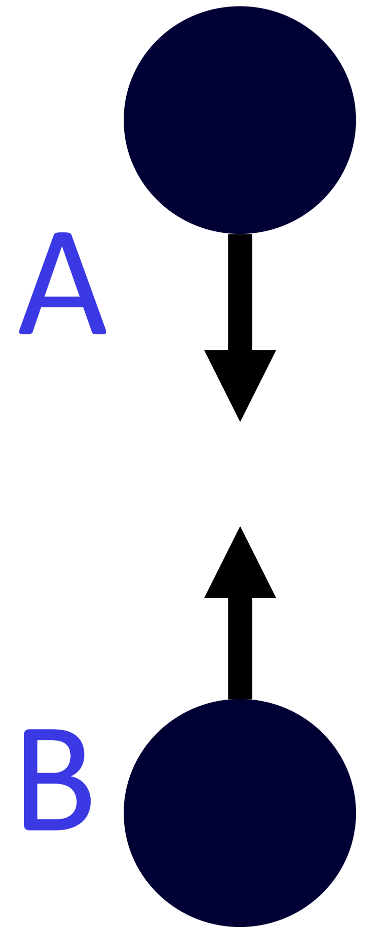
$$x = k^2(x - vt) + kvt' or \; t' = kt + \left(1 - \frac{k^2}{kv}\right)x$$

In this case, x=ct and x'=ct'

# Lorentz Transformation

- *Let's make sure to understand the transformations*
- *For a correct transformation, we make a guess based on the considerations that a single event in S must be also a single event in S', that we seek simple solutions and that when the speeds are close to "normal" we should get the Galilean before.*




At t=0, t'=0 also. Then a signal is sent, and observers in each system measure the speed with which the signal spreads out

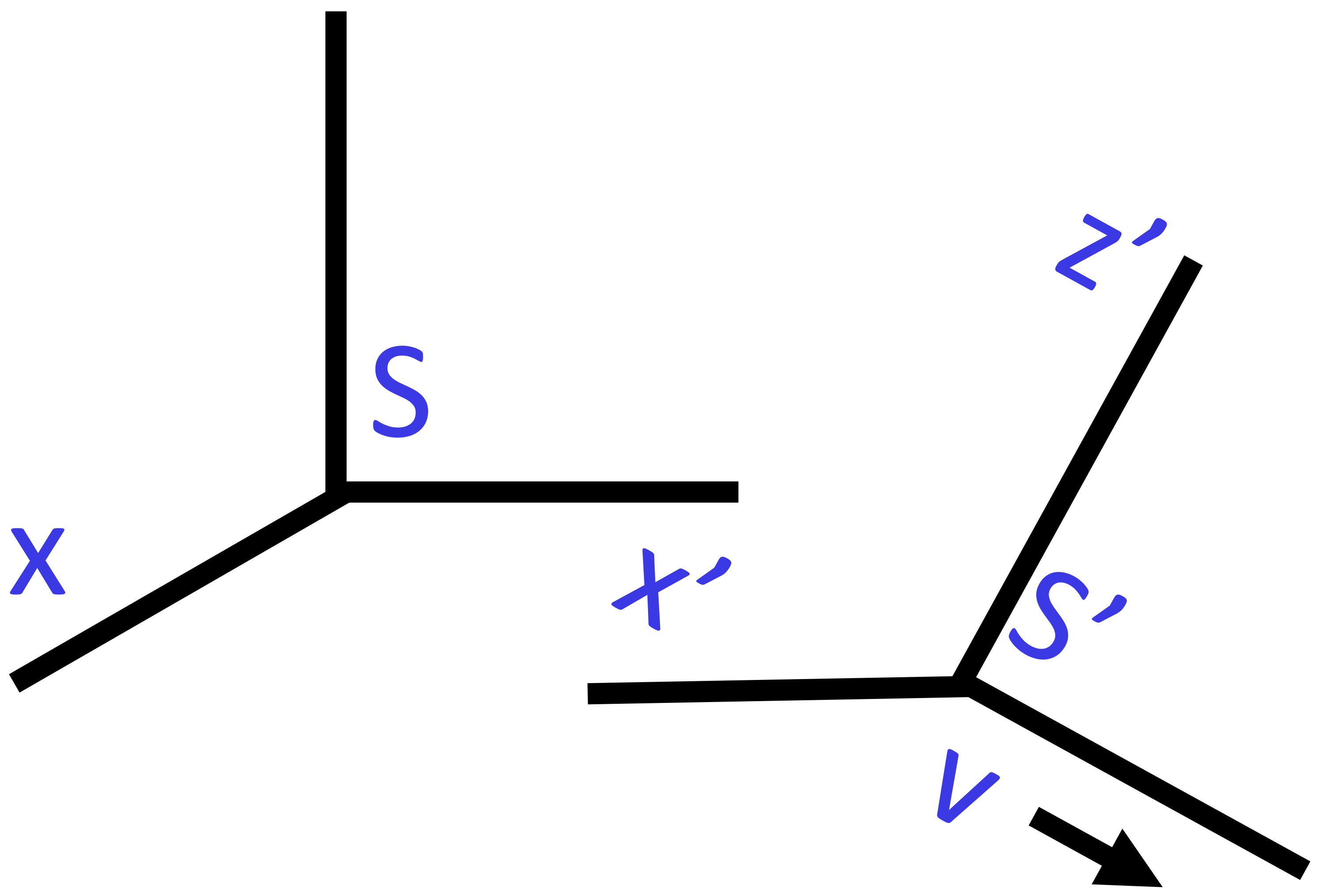
$$k(x - vt) = ckt + (1 - k^2/kv)cx$$

$$x = \frac{ckt+vkt}{k-\frac{1-k^2}{kv}c} = ct\left[\frac{k+\frac{v}{c}k}{k-\frac{1-k^2}{kv}(c)}\right] = ct\left[\frac{1+\frac{v}{c}}{1-\frac{1-k^2}{k}\left(\frac{c}{v}\right)}\right]$$

So $k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. In this case, x=ct and x'=ct'

# Spacetime

- *We can now formulate an underpinning of our visual universe*




$$s^2 = x^2 + y^2 + z^2 - (ct)^2$$
$$= x'^2 + y'^2 + z'^2 - (ct')^2$$

- *The invariance of spacetime under a Lorentz transformation is the same as describing a rotation of the S and S' coordinate systems.*

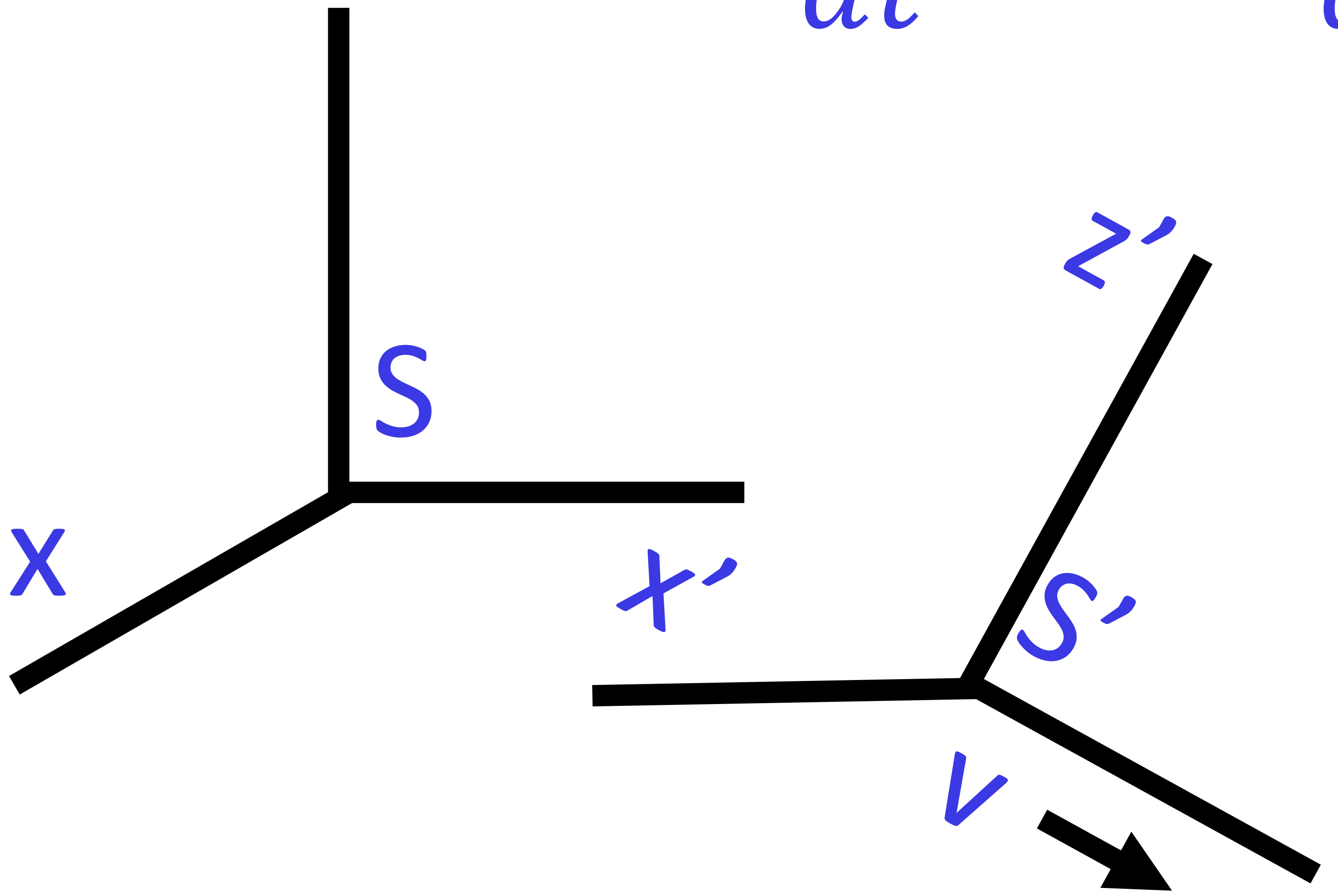
- *Similarly, the (electric) fields and (linear) momentum are invariant*

$$p_x^2 + p_y{}^2 + p_z{}^2 - \left(\frac{E}{c}\right)^2$$

# Spacetime – velocity addition

- *Suppose a car speeds down a road with its headlights on when the velocity is v. What do observers in the car and on the road see?*

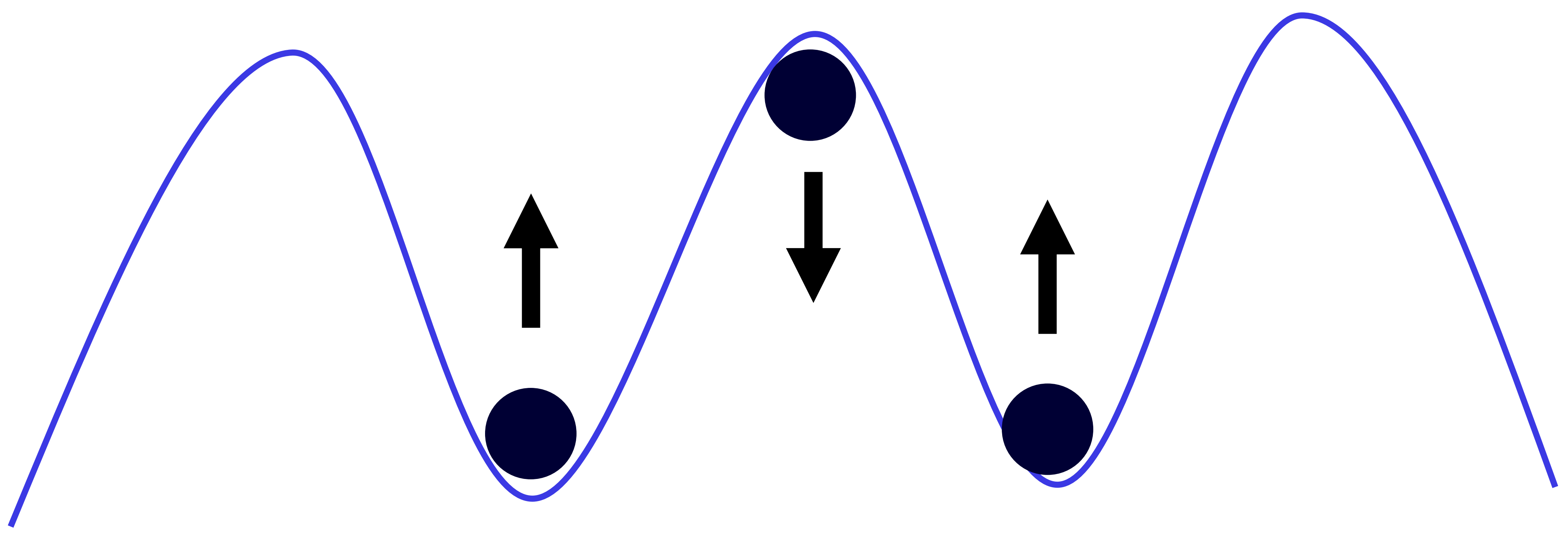
$$V_x = \frac{dx}{dt}; V_y = \frac{dy}{dt}; V_z = \frac{dz}{dt} \text{ and } V_x' = \frac{dx'}{dt'}; V_y' = \frac{dy'}{dt'}; V_z' = \frac{dz'}{dt'}$$




- *The invariance of spacetime under a Lorentz transformation is the same as describing a rotation of the S and S' coordinate systems.*
- *Similarly, the (electric) fields and (linear) momentum are invariant*

$$dx = \frac{dx' + vdt'}{\sqrt{1-\frac{v^2}{c^2}}}; dy = dy'; dz = dz'; dt = \frac{dt' + \frac{vdz'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \to V_x = \frac{V_x' + v}{1 + vV_x'/c^2} = c$$

# Classical Wave Phenomena



- In classical physics, we imagine a wave and particle as distinct from one another.
- Some waves can carry particles, or are made up of particles that move, such as the ocean water waves generated by water molecules that move upwards and downwards to generate a wave motion.
- In other waves, fields may be vibrating in space and time.
- For light, our finding is that photons can be considered the result of vibrations from electric and magnetic energy waves.

Imagine a stone thrown into a pond, generating ripples in space and time



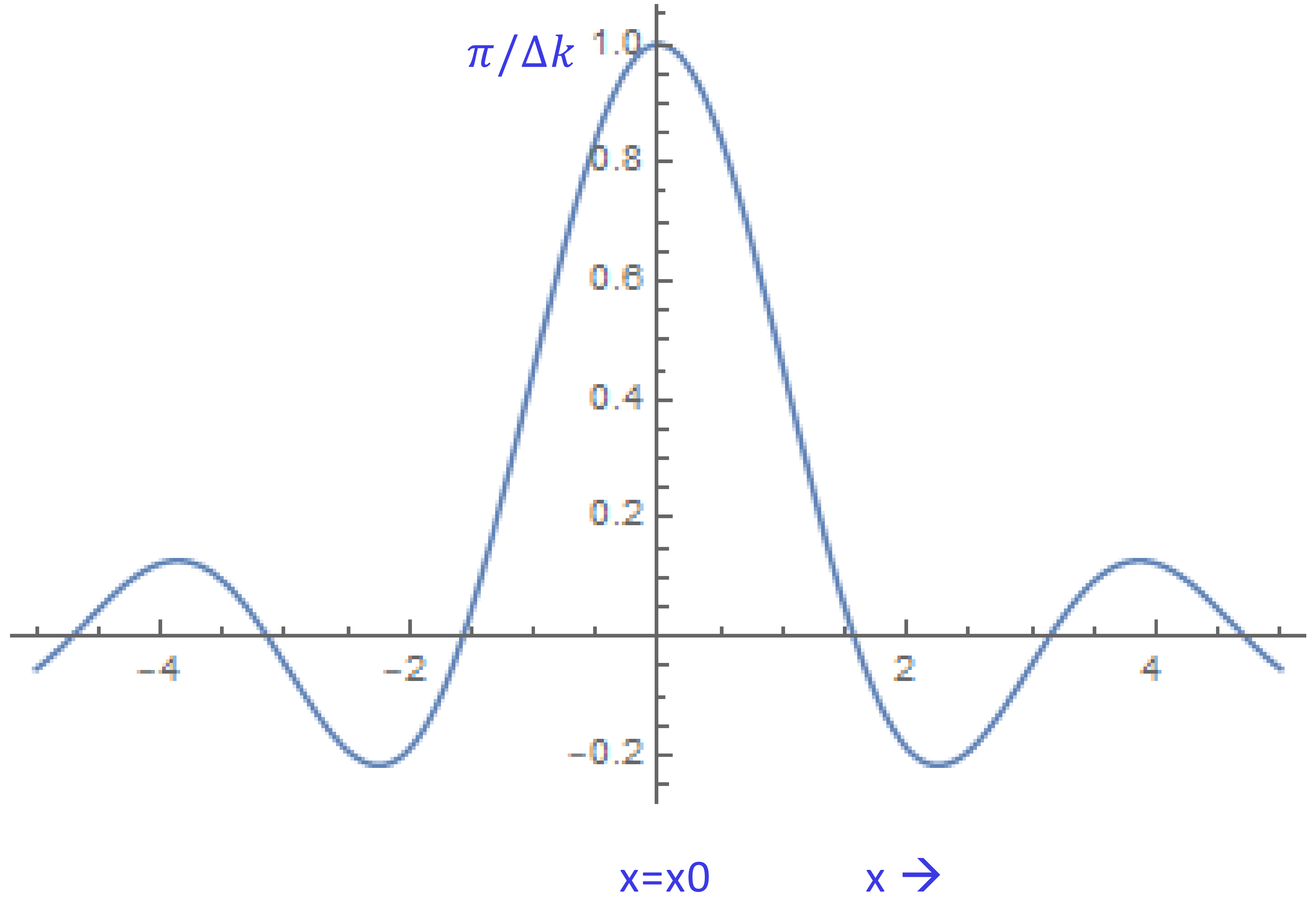
From Handbook of Chemistry & Physics

# Wave Packet: How it looks

$$F(x, x_0) = \int_{k_0-\Delta k}^{k_0+\Delta k} dk\, e^{ik(x-x_0)} = \frac{2\sin(\Delta k(x-x_0))}{x-x_0} e^{ik_0(x-x_0)}$$

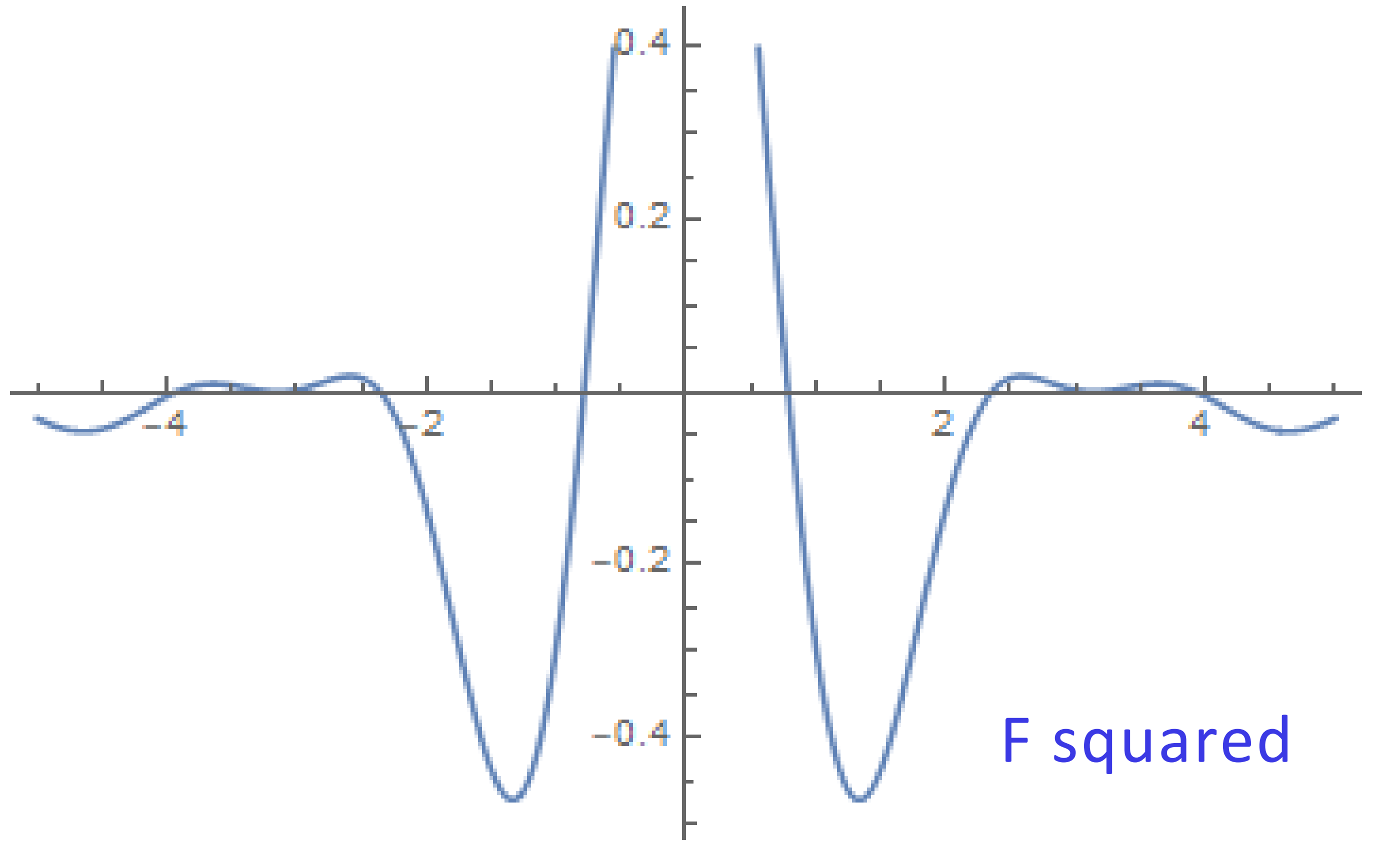
```
In[∘]:= Plot[Re[(1*Sin[x - 0] / (x - 0)) * (Exp[I*1* (x - 0)])], {x, -5, 5}]
```

Out[∘]=




```
In[∘]:= Plot[Re[(1*Sin[x - 0] / (x - 0)) * (Exp[I*1* (x - 0)]) * (1*Sin[x - 0] / (x - 0)) *
        (Exp[I*1* (x - 0)])], {x, -5, 5}]
```

Out[∘]=

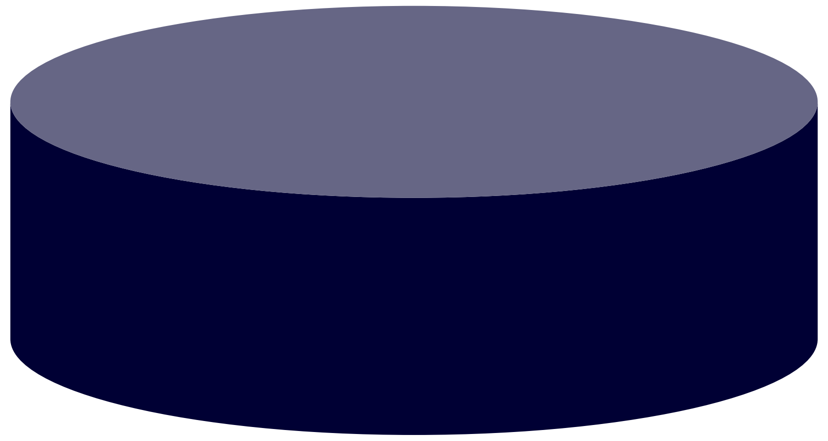


# The Blackbody Radiation and the Ultraviolet Catastrophe

- Something classical couldn't also explain.
- See in class lecture notes.

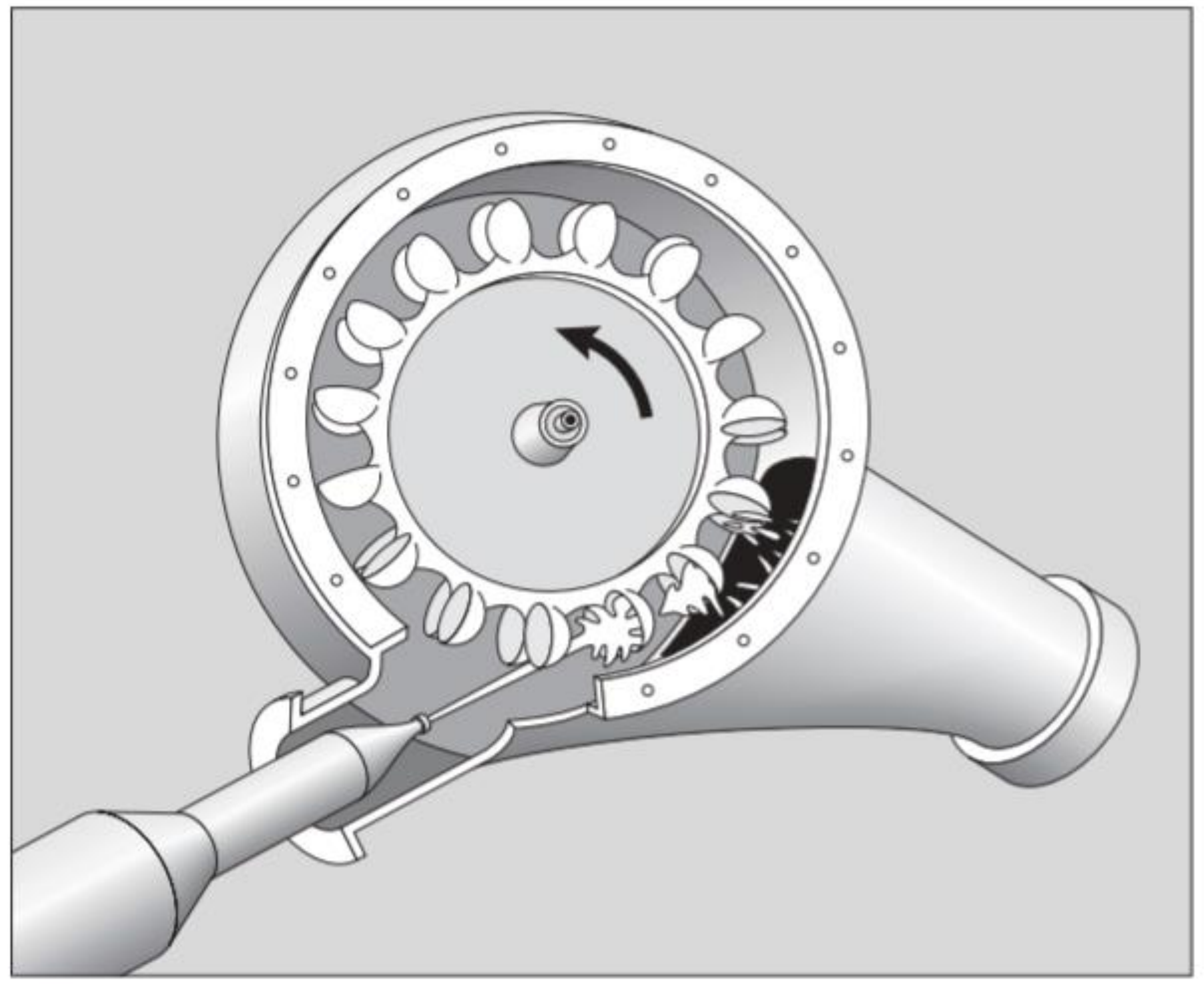
- End of Slides for Fundamental Concepts

# Generating Electricity with Captured Water



- Potential energy gain of falling water assuming 6 inch tube with 1 inch width of water (1g/cc) mgH = 221.36 [$kgm^2/s^2$=J].
- [net head (feet) × flow (gpm)] ÷ 10 = W
- An estimate of power output at 53% efficiency
- For 10 kW system with 40 ft net head, we'll need a flow of 2 gpm.
- Small turbines such as on the right which can use as little as 13 inches of water.
- Water/solution based, run pumps in reverse?


The submersible Jack Rabbit turbine was originally designed to power scientific instruments during marine oil exploration expeditions.



Pelton wheels, like this one, can be purchased with one or more nozzles. Multi-nozzle systems allow a greater amount of water to impact the runner, which can increase wheel output.

Can the flow be patterned?

Can the system be connected to a storage unit, for water and for electricity?

- Homework images