PHILLIP WU

MODERN PHYSICS

Thermodynamic Concepts, Biomagnetism, Potentials and Transport

MODERN PHYSICS

Thermodynamic Concepts

- Summary

Biomagnetism

Potentials and Transport

THE FIRST AND SECOND LAWS OF THERMODYNAMICS

$$dU = TdS + dE$$

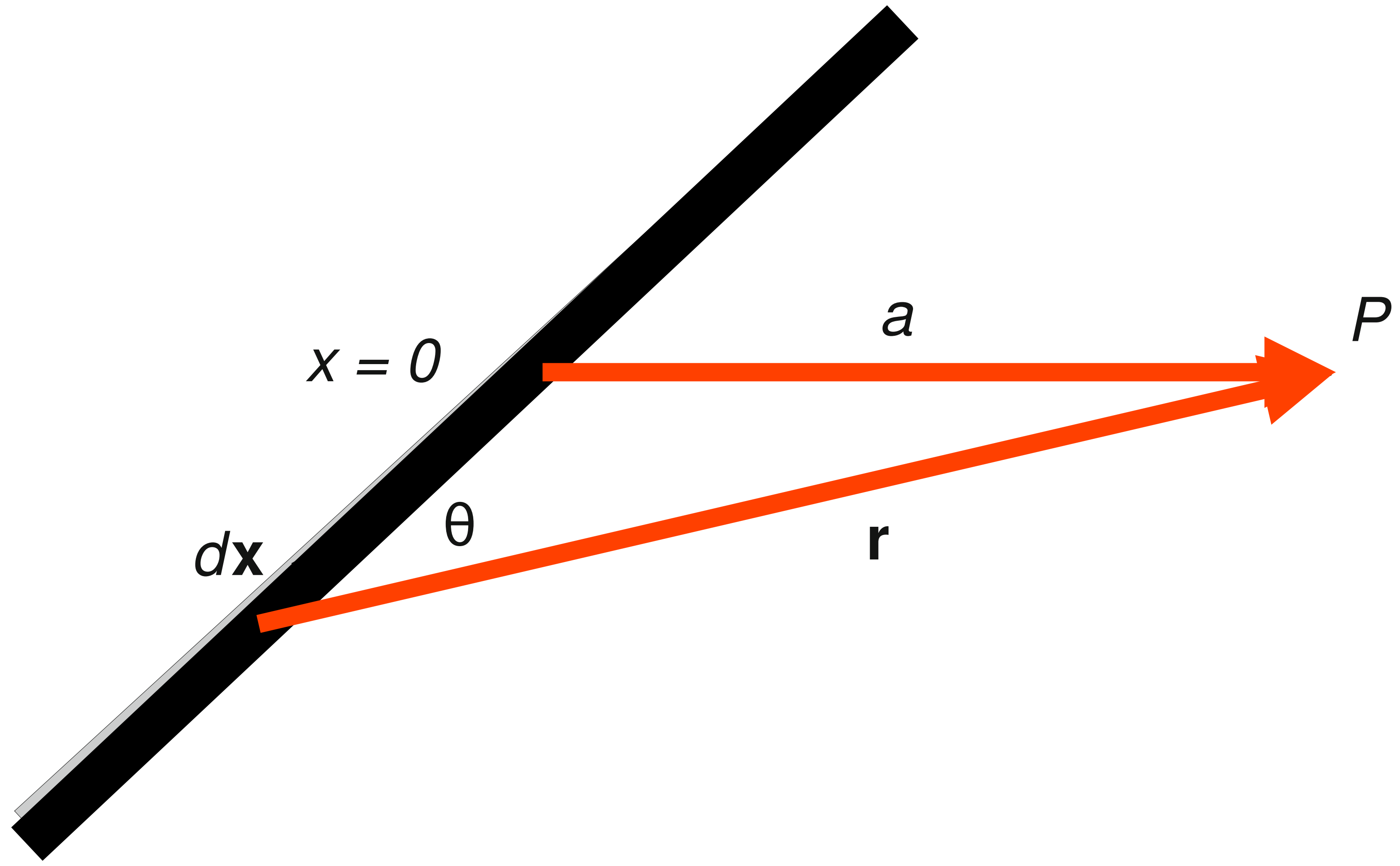
PHILLIP WU

SUMMARY OF THERMODYNAMICS AND STATISCAL MECHANICS

- If mechanical work is performed by pressure p, the energy is
- $dE = -pdV.$
- The energy is described by the Helmholtz ($F = U - TS$) or Gibbs free energy ($G = U - TS + pV$).
- In many solid state problems, the application of thermodynamics boils down to evaluating when $dF = 0$ or $dG = 0$.

BIOT-SAVART LAW

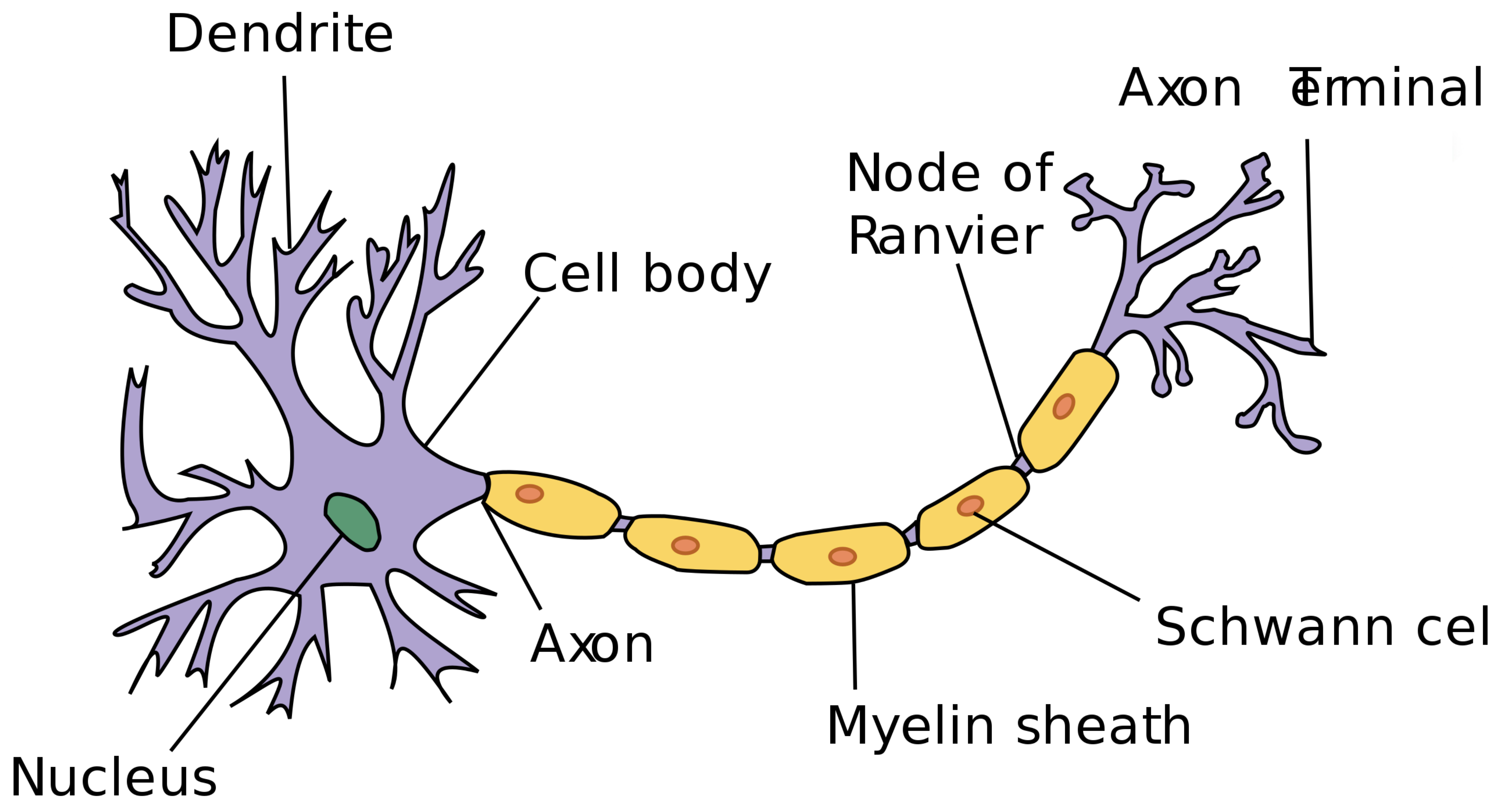
- We calculate the field at some point P away from a current carrying wire.
- $dB = \frac{\mu_0 i}{4\pi}\frac{ds \times r}{r^3}$
- $dB = \frac{\mu_0 i}{4\pi}\frac{dx sin\theta}{r^2} = \frac{\mu_0 i}{4\pi}\frac{adx}{r^3}$
- $B = \frac{\mu_0 i}{4\pi}\int \frac{adx}{(a^2 + x^2)^{3/2}} = \frac{\mu_0 ia}{4\pi}[\frac{x}{a^2(x^2 + a^2)^{1/2}}]_{-\infty}^{\infty} = \frac{\mu_0 i}{2\pi a}$



Right hand rule!

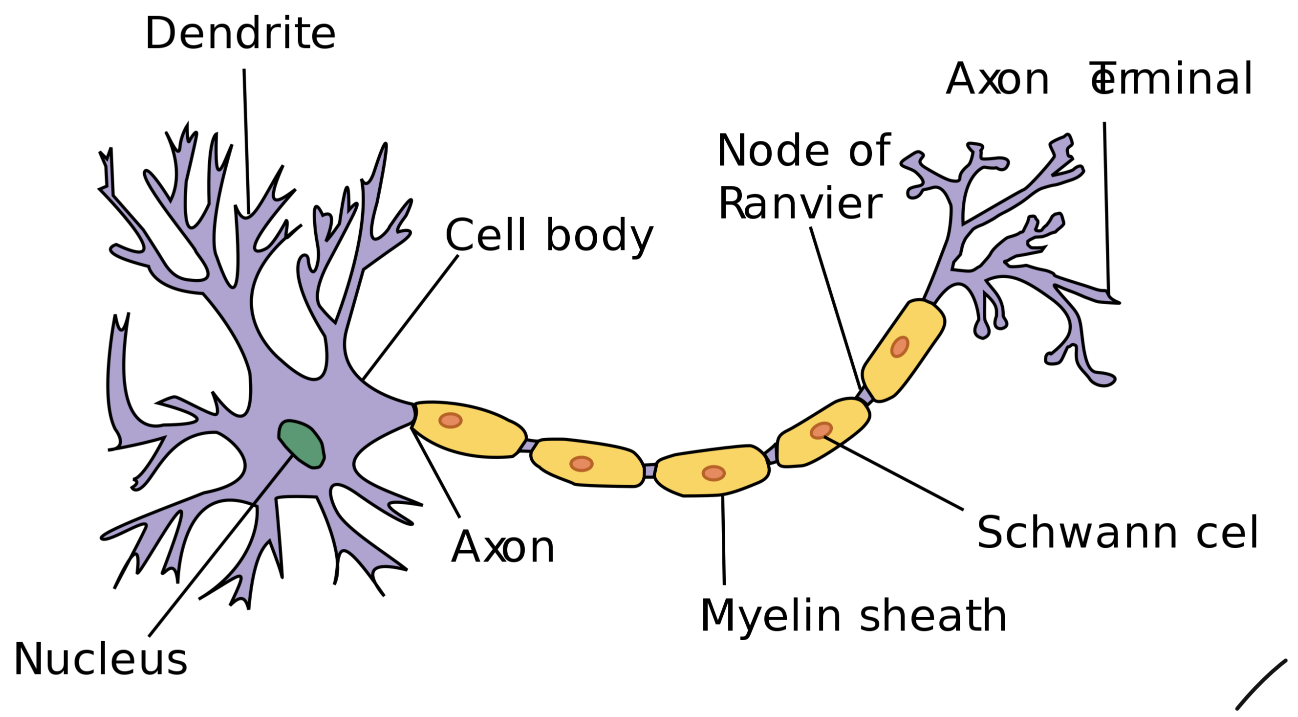
BIOMAGNETISM: BIOT-SAVART LAW APPLIED TO AXONS

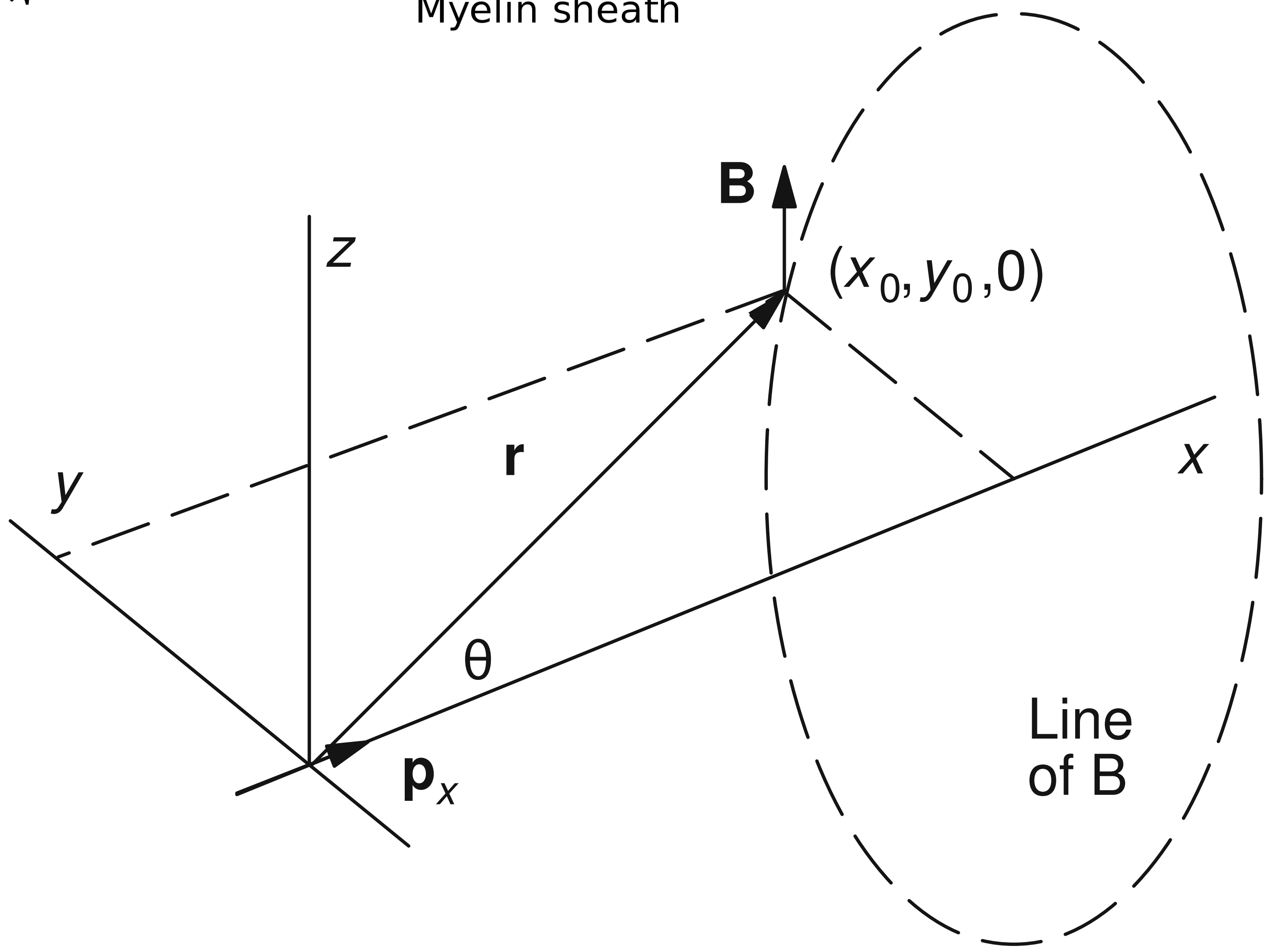
- We calculate the field at some point P away from a current carrying wire.
- $dB = \frac{\mu_0 i}{4\pi}\frac{ds \times r}{r^3}$
- $dB = \frac{\mu_0 i}{4\pi}\frac{dx sin\theta}{r^2} = \frac{\mu_0 i}{4\pi}\frac{adx}{r^3}$
- $B = \frac{\mu_0 i}{4\pi}\int\frac{adx}{(a^2+x^2)^{3/2}} = \frac{\mu_0 ia}{4\pi}[\frac{x}{a^2(x^2+a^2)^{1/2}}]_{-\infty}^{\infty} = \frac{\mu_0 i}{2\pi a}$



BIOMAGNETISM: BIOT-SAVART LAW APPLIED TO AXONS

- **We calculate the field at some point P away from a current carrying wire.**
- $$ds \times r = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dx & 0 & 0 \\ x_0 - x & y_0 & 0 \end{vmatrix} = dx * y_0 * \hat{z}$$
- $$B = \frac{\mu_0 y_0}{4\pi} \int \frac{i_i(x)dx}{[(x_0 - x)^2 + y_0^2]^{3/2}} = \frac{\mu_0 a^2 \sigma_i y_0}{4} \int \frac{[dv_i(x)/dx]dx}{[(x_0 - x)^2 + y_0^2]^{3/2}}$$
- **Here the current** $i_i = -\pi a^2 \sigma_i (dv_i/dx)$

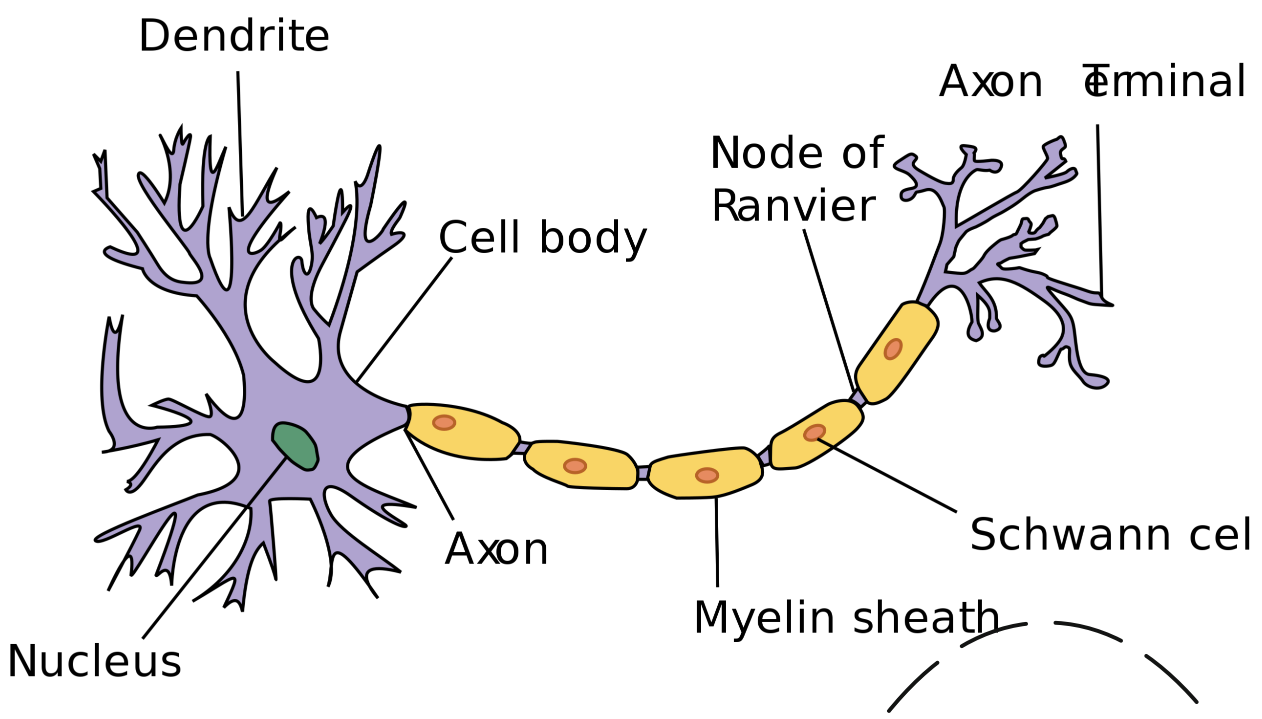


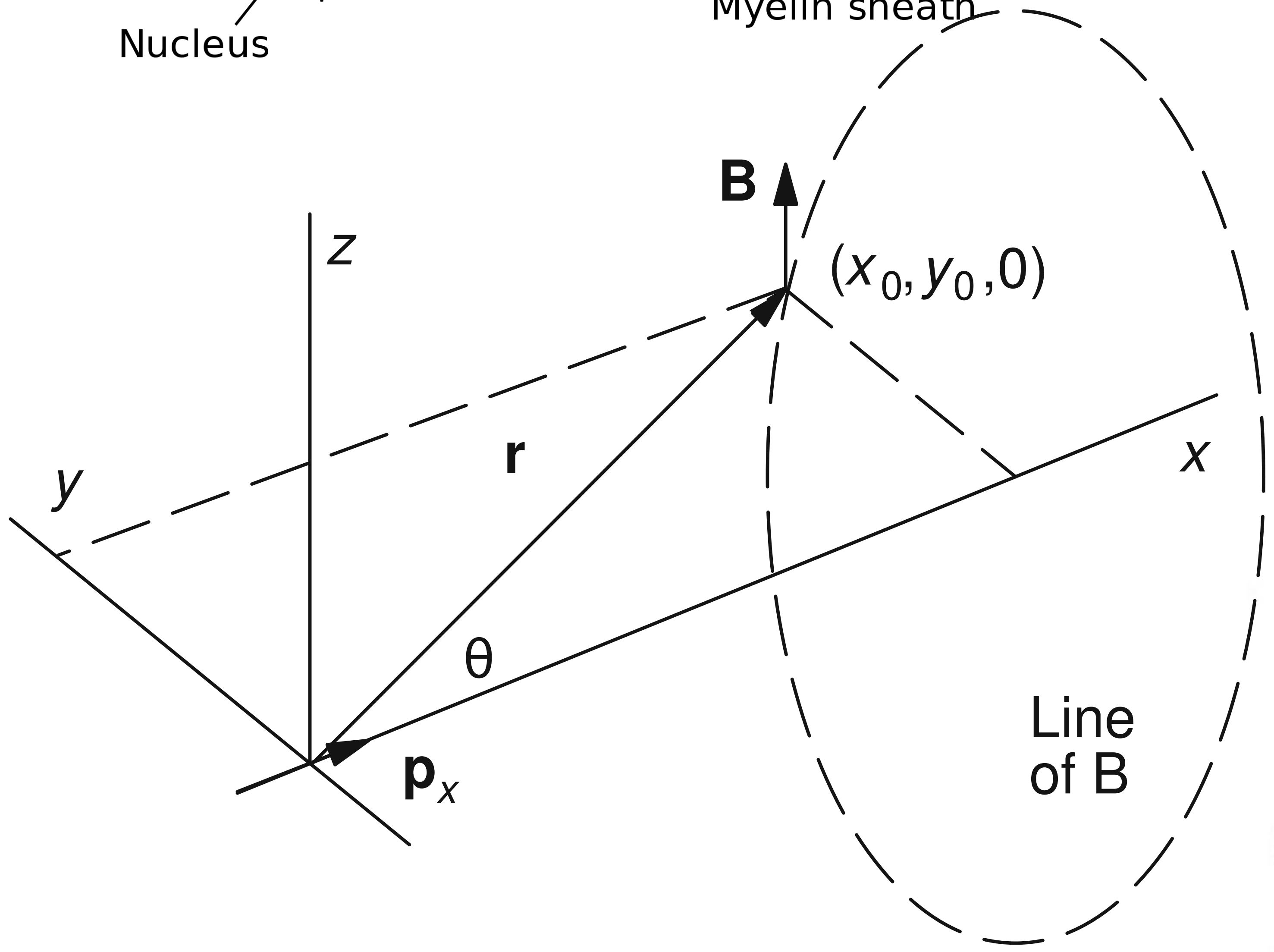


Current element idx or current dipole p_x stretched along x-axis

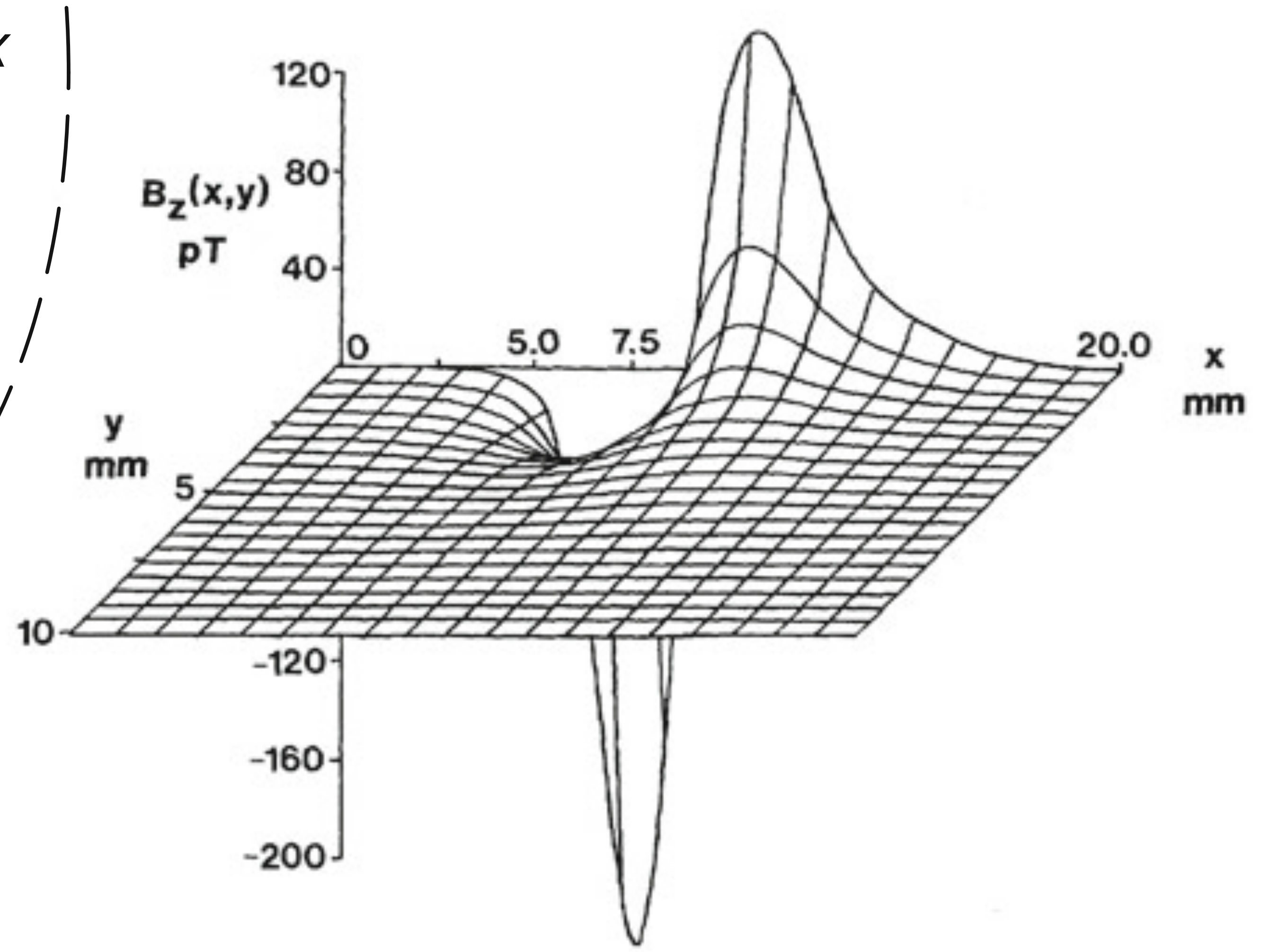
BIOMAGNETISM: BIOT-SAVART LAW APPLIED TO AXONS

- We calculate the field at some point P away from a current carrying wire.
- $$ds \times r = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dx & 0 & 0 \\ x_0 - x & y_0 & 0 \end{vmatrix} = dx * y_0 * \hat{z}$$
- $$B = \frac{\mu_0 y_0}{4\pi} \int \frac{i_i(x)dx}{[(x_0 - x)^2 + y_0^2]^{3/2}} = \frac{\mu_0 a^2 \sigma_i y_0}{4} \int \frac{[dv_i(x)/dx]dx}{[(x_0 - x)^2 + y_0^2]^{3/2}}$$
- Here the current $i_i = -\pi a^2 \sigma_i (dv_i/dx)$



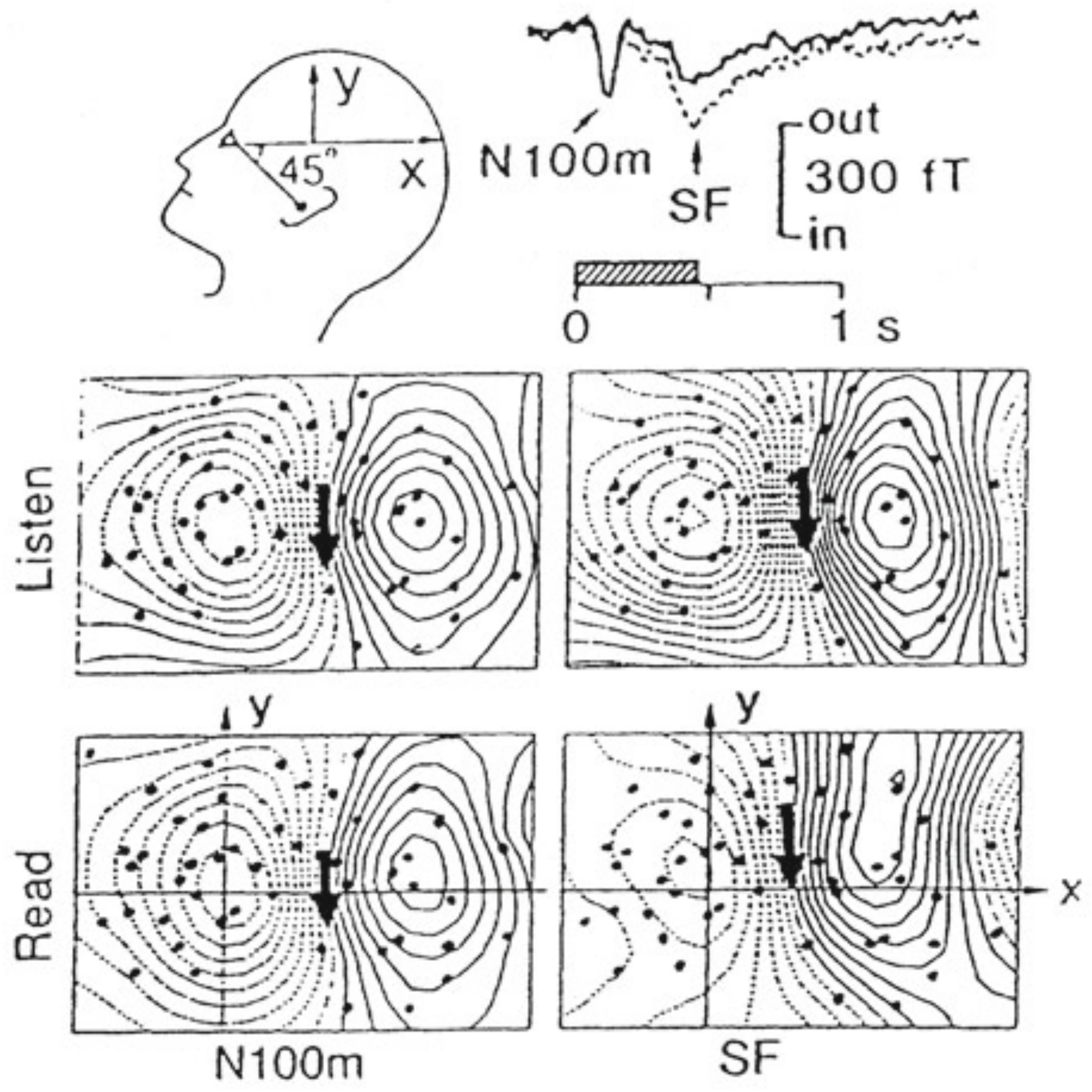


Current element idx or current dipole p_x stretched along x-axis



BIOMAGNETISM: BIOT-SAVART LAW APPLIED TO AXONS

- The figure shows magnetic field maps recorded over the scalp of a subject who heard a series of words and either ignored them by reading something else or listened carefully and counted how many of the words were in a predetermined list (from Haamaalaainen et al. 1993).
- The second sustained field peak is stronger in subjects that were focused with paying attention to the list.



BIOMAGNETISM

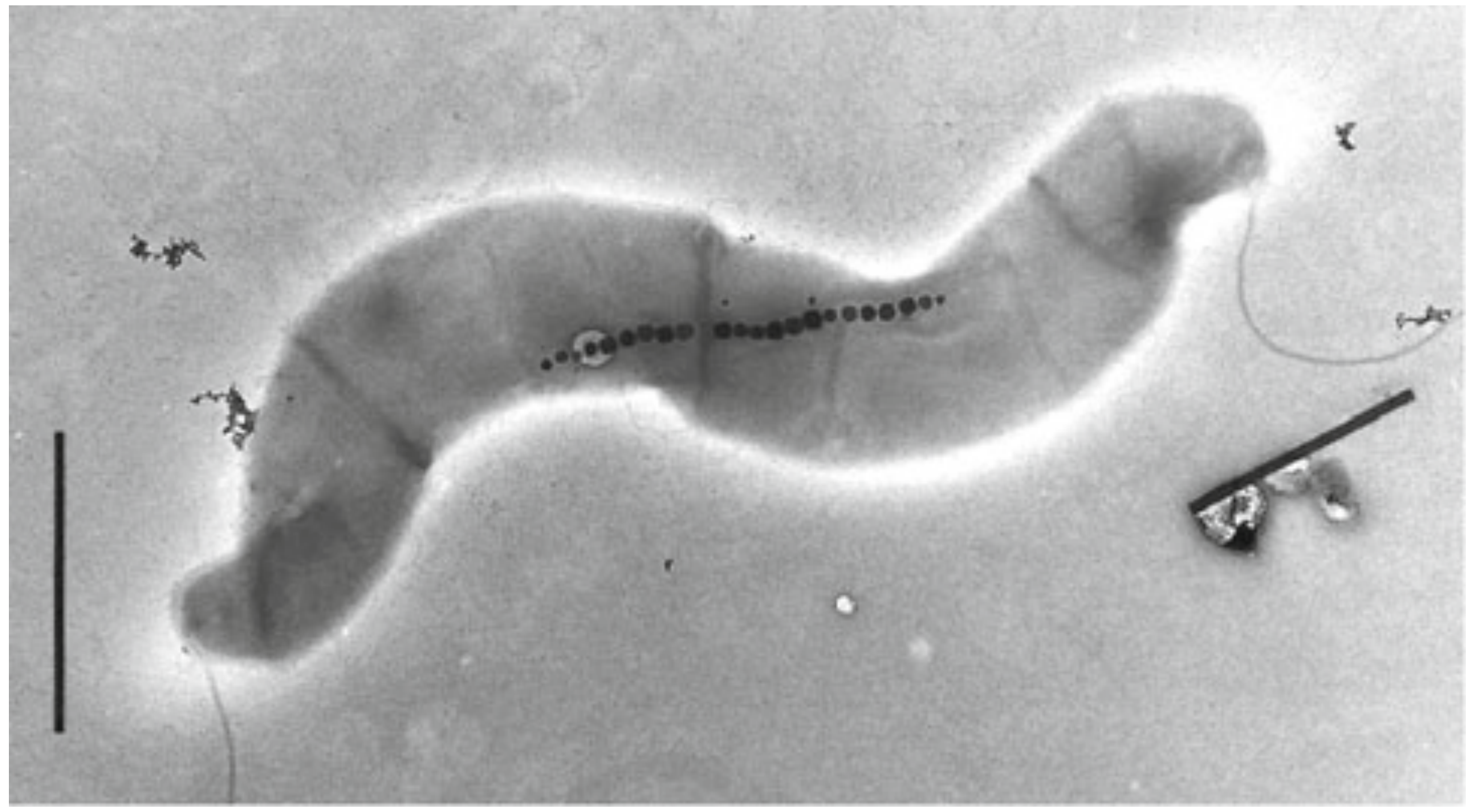


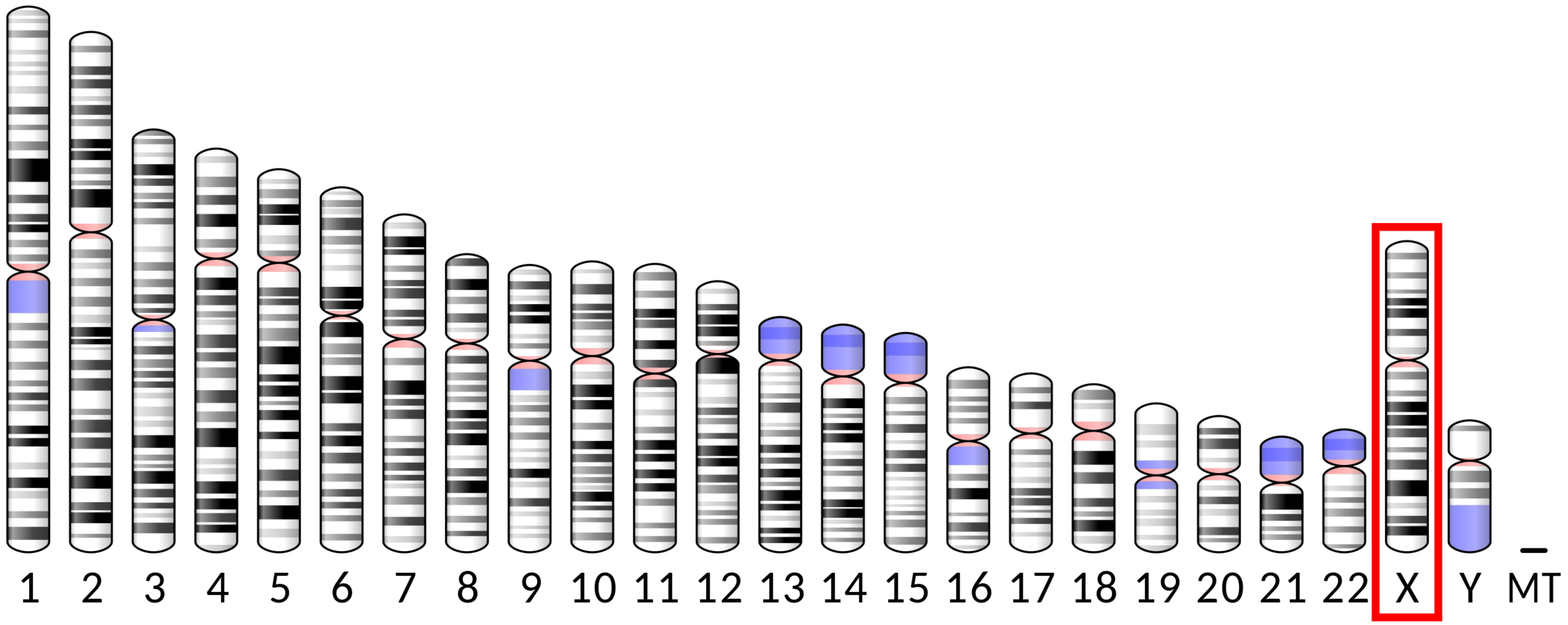
Fig. 8.25 The small black dots are magnetosomes, small particles of magnetite in the magnetotactic bacterium *Aquaspirillum magnetotacticum*. The vertical bar is 1 μm long. The photograph was taken by Y. Gorby and was supplied by N. Blakemore and R. Blakemore, University of New Hampshire.

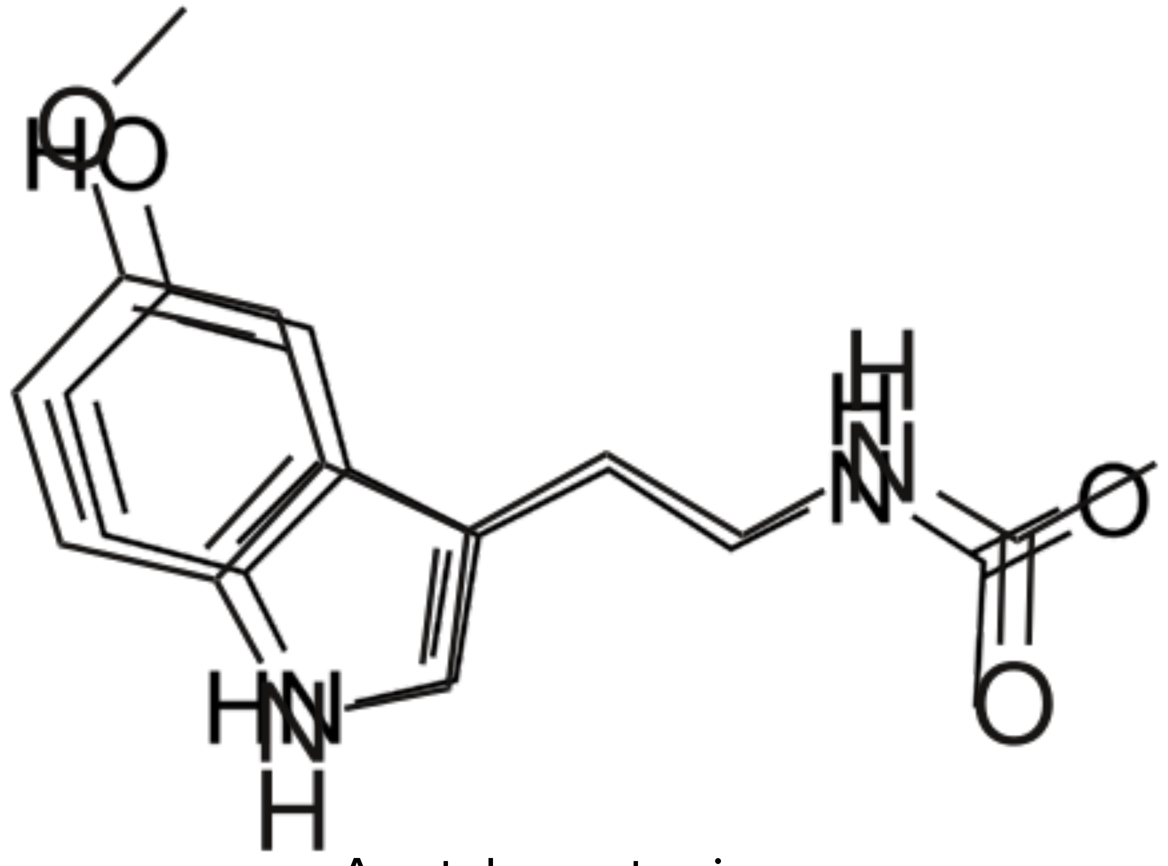
The size distribution of these particles averages in many cases around 50 nm, important for maintaining the remnant magnetization field.

- Magnetic properties of organisms, such as algae, worms, and birds allow them to geospatially geolocate.
- Certain bacteria live in oxygen-deficient, sulfur-rich environments, thus contain more Fe_3S_4 (~600 K) instead of magnetite Fe_3O_4 (Curie 847 K).
- In medical research, there are reports of single domain magnetite 10 - 70 nm in diameter used to attack cancerous cells with hyperthermia (effectively heating the cells when an external oscillating field is applied, which causes the nanoparticles to rotate).
- The body contains, by some sources, 3 - 4 g of iron, mostly stored in the liver.

BIOMAGNETISM

- In birds, the pineal gland is likely to be magneto sensitive. The "mechanism of the pineal's response is a decrease in enzymatic activity of hydroxyindole-O-methyl-transferase (HIOMT) and N-acetyl-serotonin- transferase (NAT) when the animal is exposed to a 50% decrease in the ambient magnetic field" (Beason, Semm).
- In humans, this enzyme (acetyl-serotonin $C_{12}H_{14}N_2O_2$) is encoded by the gene near the end-caps of the X-chromosome, and is part of the pathway of conversion of normelatonin to melatonin (an important part regulating sleep-wake cycle, and interaction with melanin, which changes skin color).





Acetyl-serotonin

https://en.wikipedia.org/wiki/Acetylserotonin_O-methyltransferase

https://en.wikipedia.org/wiki/Melatonin

BIOMAGNETISM OF THE HUMAN BODY

- A simple analysis to show that energetically, using magnetic effects can be meaningful: assume a bio-magnetosome (some biomagnetic object) has an energy in the Earth's field of mB. Compare to the thermal energy, this factor is
- $\frac{mB_{Earth}}{k_B T} = \frac{(6.4 * 10^-17 J/T)(5 * 10^-5 T)}{(1.38 * 10^{-23} J/K)(300K)} = 0.77$. For larger magnetosomes, approximate magnetization at 100 times larger, then this ratio approaches 20. The magnetic field due to a typical power line is 100 times smaller, at $5*10^{-7}$ T!

- For electric fields, the situation is not so simple, due to our skin and dielectric effects which work to attenuate the electric field strength. Nonetheless, it is still possible in some to measure a "resistance" with a handheld multimeter.

BIOELECTRICITY OF THE HUMAN BODY

- Let's consider two simple models: (1) an infinite slab of tissue with dielectric constant (here written as) κ and electrical conductivity σ. Gauss' Law gives the charge at the surface:
- $-\epsilon_0 E_0 cos\omega t + \kappa\epsilon_0 E_1(t) = \sigma_q(t)$
- $\dfrac{dE_1}{dt} + \dfrac{\sigma}{\kappa\epsilon_0}E_1 = -\dfrac{\omega}{\kappa}E_0 sin\omega t \rightarrow A sin\omega t + B cos\omega t$ as solutions to E_1.
- We solve for the coefficients of this solution as $A = -\dfrac{\omega\tau_t}{\kappa(1+\omega^2\tau_t^2)}E_0 \approx -\dfrac{\omega\epsilon_0}{\sigma}E_0$
- $B = -\omega\tau_t A = -\dfrac{(\omega\tau_t)^2}{\kappa(1+\omega^2\tau_t^2)}E_0 \approx 0$
- Importantly, assume 60 Hz, dielectric constant 10⁶, membrane response time of 1 microsecond, we arrive at $E_1 \approx A \approx 33 * 10^{-9} E_0$ (a tiny number).

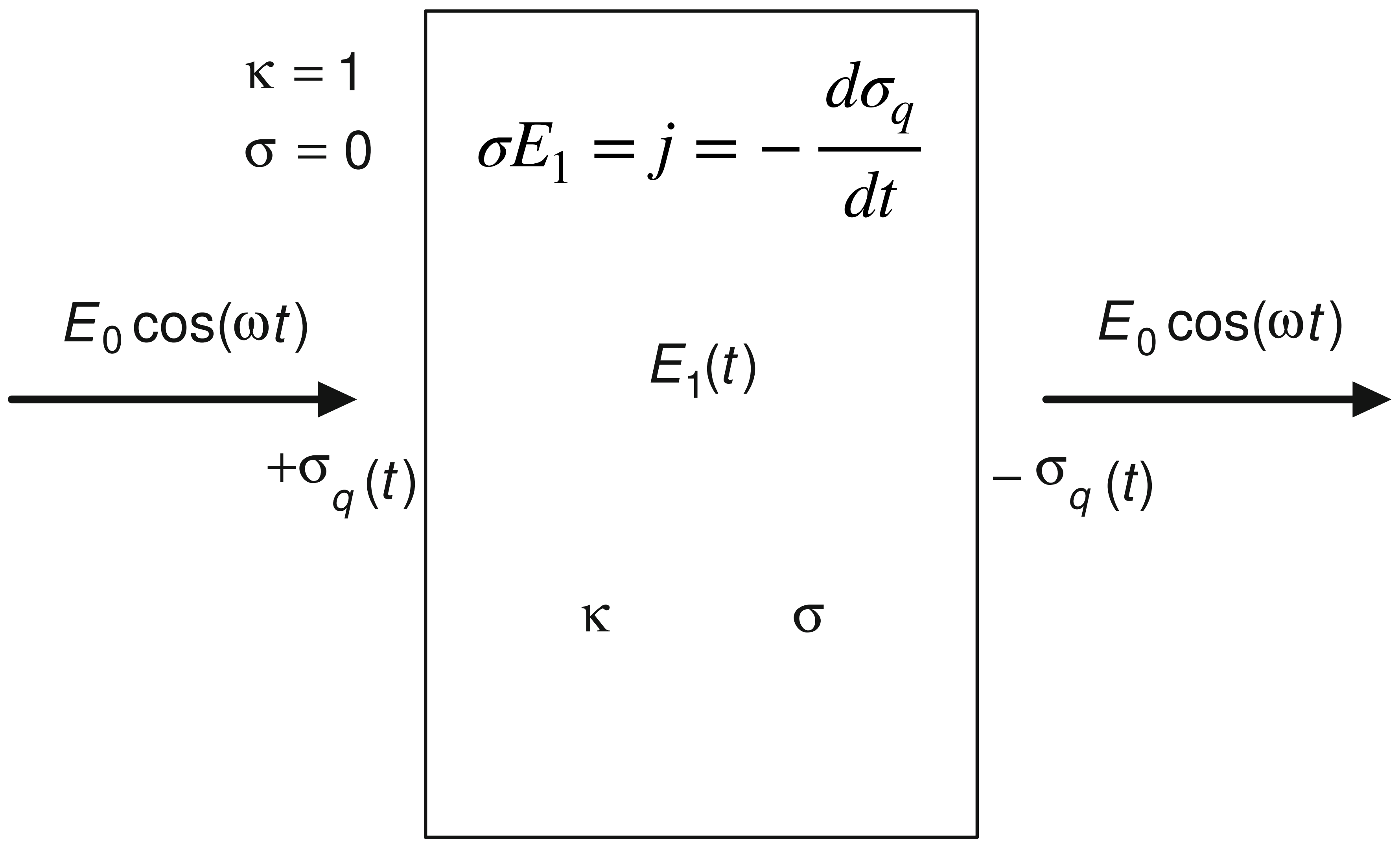


Table 9.5 Comparison of the signal in a cell to thermal noise for an applied electric field in air $E_0 = 300$ V m^{-1}. From Eq. 9.71, $E_1 = 10^{-5}$ V m^{-1}. $T = 300$ K. $z = 10$. $d = 10^{-5}$ m

Model	Outside the cell	In the cell membrane	Inside the cell
E (V m^{-1})	1.0×10^{-5}	1.62×10^{-2}	5.40×10^{-10}
$k_B T/eE$ (m)	2.57×10^{3}	1.59	4.79×10^{7}
$zeEd/k_B T$	3.9×10^{-8}	6.3×10^{-5}	2.1×10^{-12}

BIOEL… MAN BODY

- Let's consi… tissue with
  dielectric c… nductivity σ.
  Gauss' Law…
- $-\epsilon_0 E_0 cos\omega$…
- $\frac{dE_1}{dt} + \frac{\sigma}{\kappa\epsilon_0}$… lutions to E_1.
- We solve f…
  $A = -\frac{\ldots}{\kappa(1\ldots}$
- $B = -\omega\tau_t A$…
- Importantl… mbrane
  response ti… $33 * 10^{-9}E_0$
  (a tiny num…

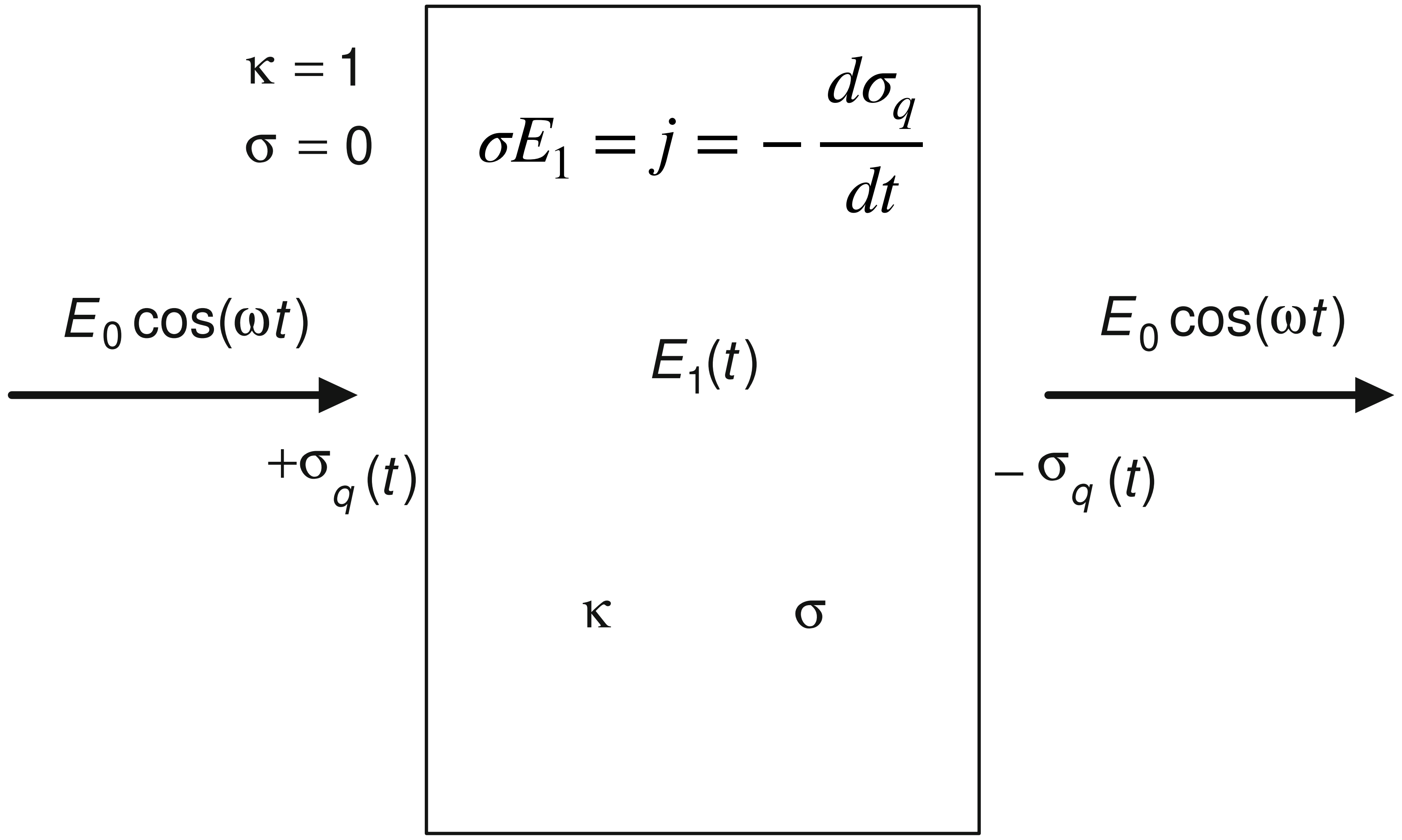
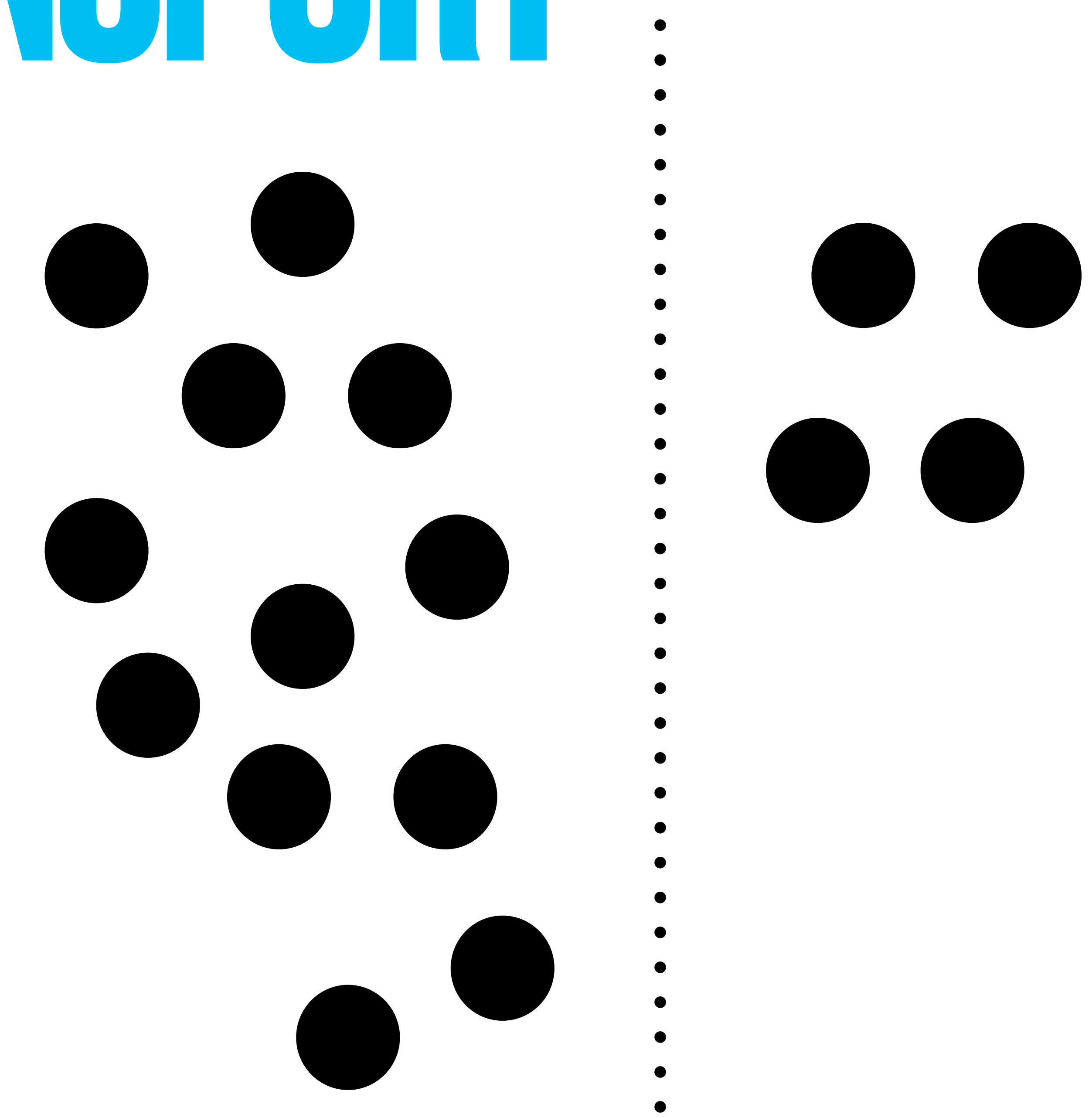


Table 9.5 Comparison of the signal in a cell to thermal noise for an applied electric field in air $E_0 = 300$ V m^{-1}. From Eq. 9.71, $E_1 = 10^{-5}$ V m^{-1}. $T = 300$ K. $z = 10$. $d = 10^{-5}$ m

Model	Outside the cell	In the cell membrane	Inside the cell
E (V m^{-1})	1.0×10^{-5}	1.62×10^{-2}	5.40×10^{-10}
k_BT/eE (m)	2.57×10^{3}	1.59	4.79×10^{7}
$zeEd/k_BT$	3.9×10^{-8}	6.3×10^{-5}	2.1×10^{-12}

DIFFUSION: POTENTIALS AND TRANSPORT

- The diffusion equation looks like this: $j_x = -D\frac{\partial C}{\partial x}$
- Generally known as Fick's First Law, with units of / m^2s for the current
- If $\partial C/\partial x = 0$, no diffusion.
- If not, then movement of particles (ions) occurs from higher concentration to lower concentration.
- Fick's law is one of many forms of transport equations.

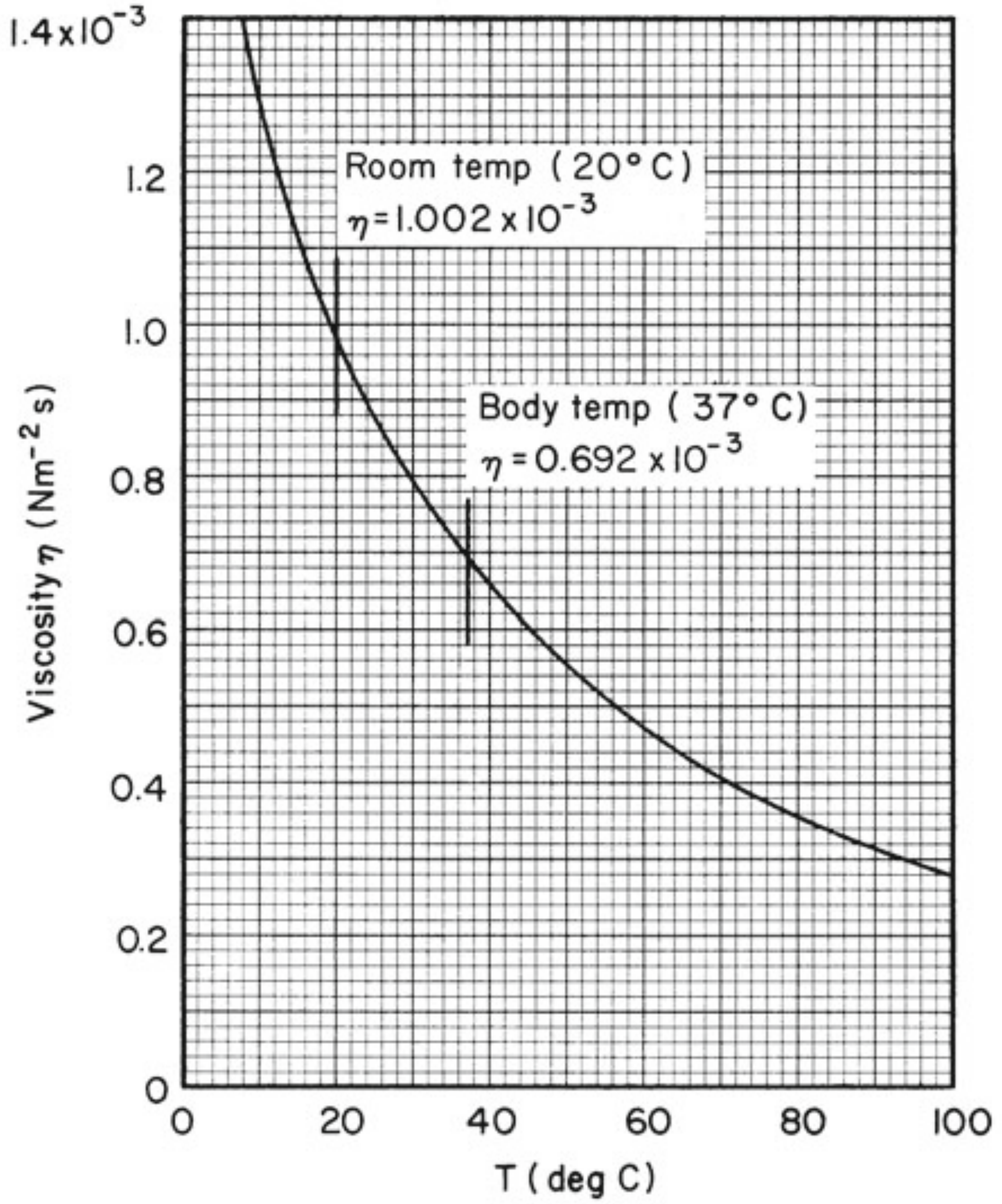


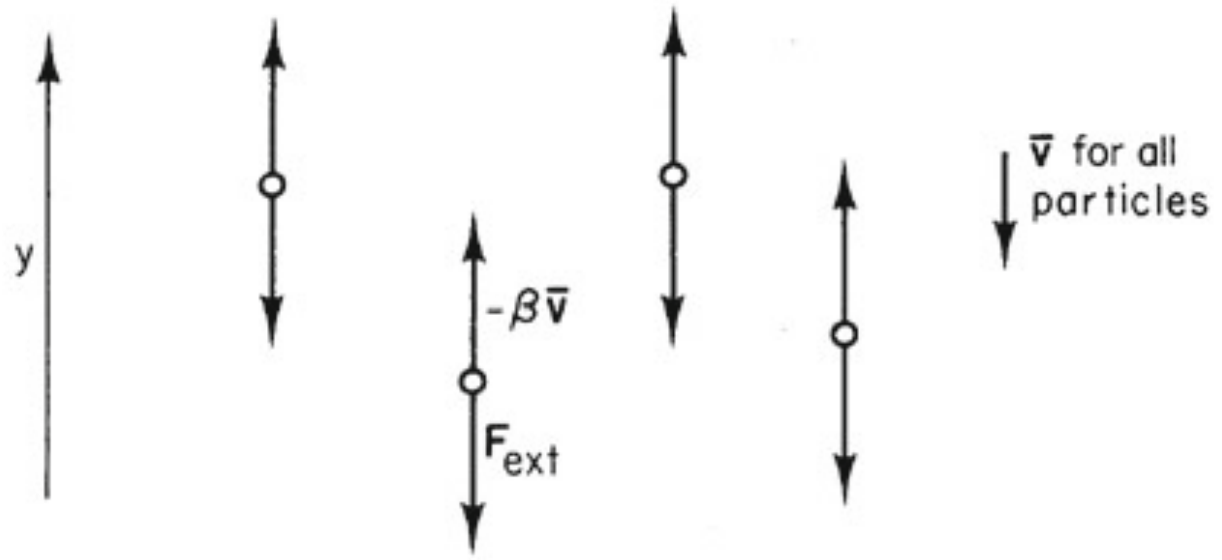
DIFFUSION: POTENTIALS AND TRANSPORT

Substance flowing	Equation	Units of j	Units of the constant
Particles	$j_s = -D\,\dfrac{\partial C}{\partial x}$	$\mathrm{m^{-2}\ s^{-1}}$	$\mathrm{m^{2}\ s^{-1}}$
Mass	$j_m = -D\,\dfrac{\partial \rho}{\partial x}$	$\mathrm{kg\ m^{-2}\ s^{-1}}$	$\mathrm{m^{2}\ s^{-1}}$
Heat	$j_H = -\kappa\,\dfrac{\partial T}{\partial x}$	$\mathrm{J\ m^{-2}\ s^{-1}}$ or $\mathrm{kg\ s^{-3}}$	$\mathrm{J\ K^{-1}\ m^{-1}\ s^{-1}}$
Electric charge	$j_e = -\sigma\,\dfrac{\partial V}{\partial x}$	$\mathrm{C\ m^{-2}\ s^{-1}}$	$\mathrm{C\ m^{-1}\ s^{-1}\ V^{-1}}$ or $\Omega^{-1}\ \mathrm{m^{-1}}$
Viscosity (y component of momentum transported in the x direction)	$j_p = -\eta\,\dfrac{\partial v_y}{\partial x}$	$\mathrm{N\ m^{-2}}$ or $\mathrm{kg\ m^{-1}\ s^{-2}}$	$\mathrm{kg\ m^{-1}\ s^{-1}}$ or Pa s

DIFFUSION: POTENTIALS AND TRANSPORT

- An example to solve the Fick's Law.
- A downward flux density, from the forces $F_{ex} - \beta\bar{v} = 0$, means that the number of particles crossing area S in Δt will be those within the cylinder of height $\bar{v}\Delta t$. The concentration then is $S\bar{v}\Delta t$
- $j_{drift} = -\bar{v}C(y)\hat{y}$.
- $j_{diff} = -D\frac{\partial C}{\partial y}\hat{y}$
- Equating these two means the system is then at equilibrium, gives
- $C(y) = C(0)e^{-F_{ext}y/k_BT}$
- From this, we find that the diffusion constant and the velocity are related, $\bar{v} = DF_{ext}/k_BT$
- So far, the solutions only require the the velocities be small enough so that a linear approximation for Fick's law and viscous forces are valid. If the diffusing particles are large enough to allow Stoke's law, then $\beta = 6\pi\eta a$ with $D = \frac{k_BT}{6\pi\eta a}$.





Particles drifting under the influence of a downward force F_{ext}

DIFFUSION: POTENTIALS AND TRANSPORT

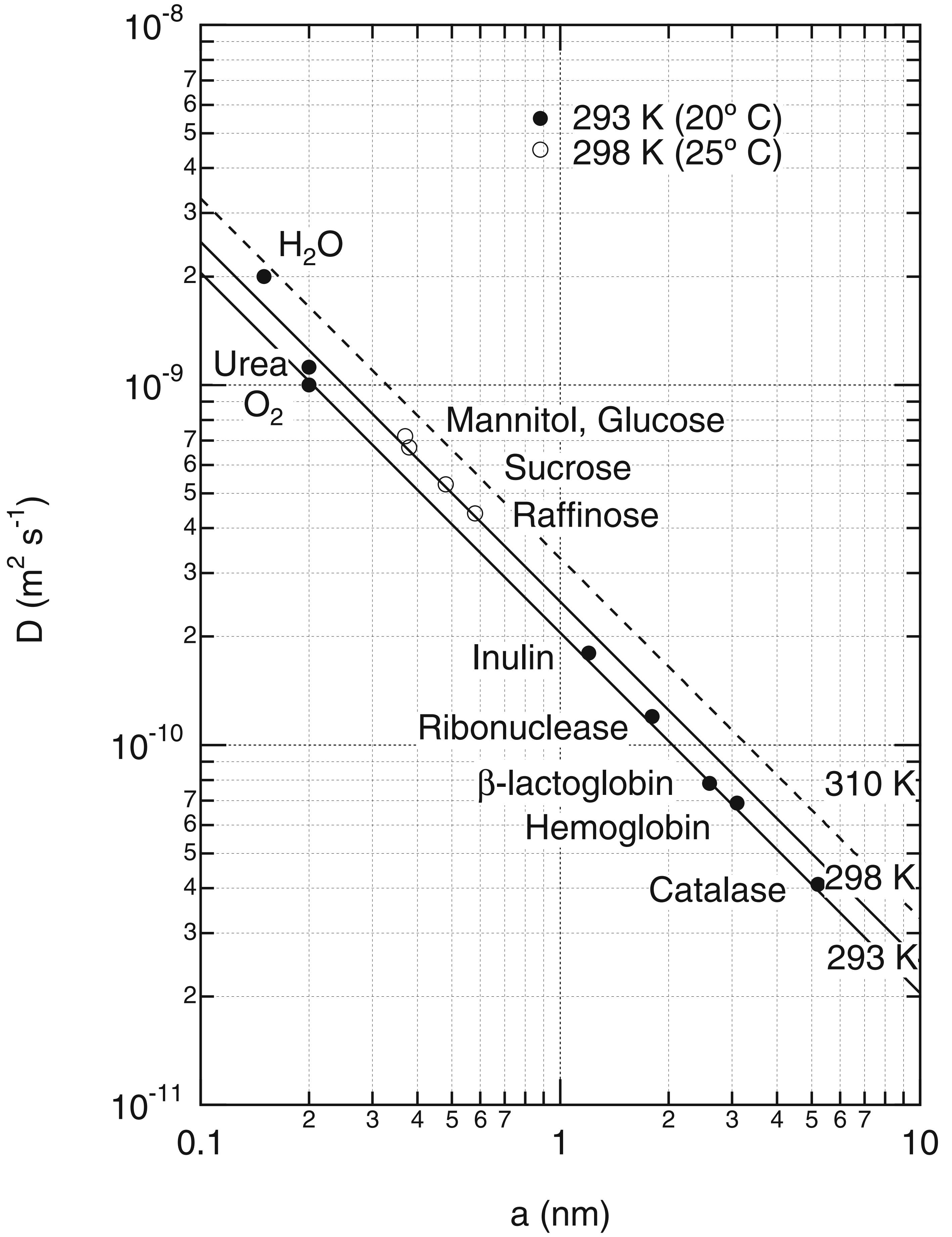


Fig. 4.11 Diffusion constant versus sphere radius *a* for diffusion in water at three different temperatures. Experimental data at 20 °C (293 K) are from Benedek and Villars (2000, Vol. 2, p. 122). Data at 25 °C (298 K) are from Weast (1972, p. F-47)

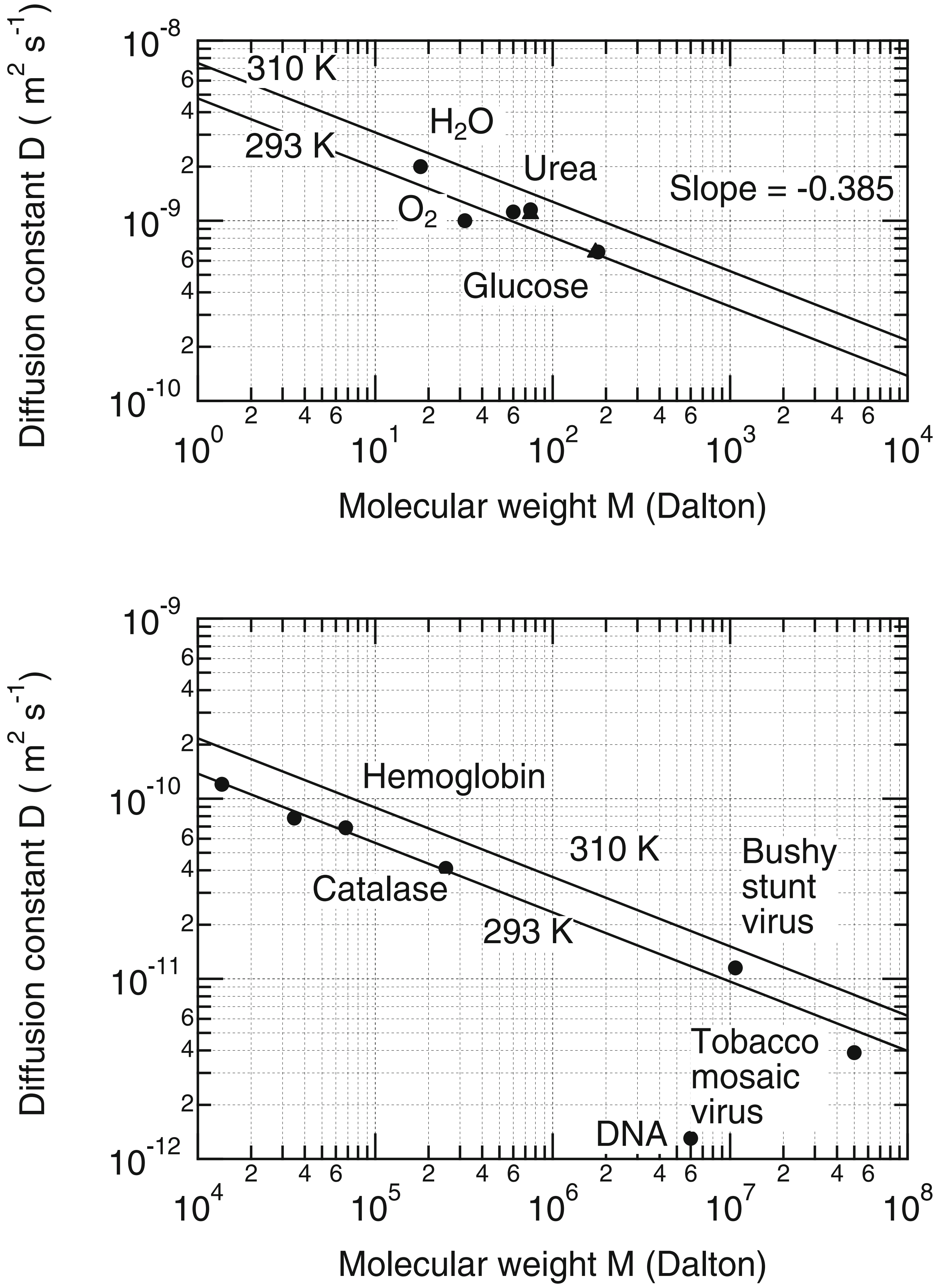


Fig. 4.12 Diffusion constant versus molecular weight in daltons. (One dalton is the mass of one hydrogen atom.) Data at 293 K are from Benedek and Villars (2000, Vol. 2, p. 122). The 293-K solid line was drawn by eye through the data; the line at 310 K was drawn parallel to it using the temperature change in Eq. 4.23. Data scatter around the line by about 30%, with occasional larger departures

DIFFUSION: POTENTIALS AND TRANSPORT

- The diffusion equation looks like this: $j_x = -D\frac{\partial C}{\partial x}$
- We can use this equation to understand ion movement in solutions, for example, where the solute particles move by diffusion. The average velocity of these particles is obtained from either being at rest with respect to a moving solution, i.e. solvent drag, or having an external force such as gravity or an electric field dragging the solute particles in the solution.
- This adds a term to the diffusion equation, $j_x = -D\frac{\partial C}{\partial x} + CV_{solut}$.
- If an external force $F = zeE$ acts on the particles, the velocity $V_{solut} - V_{solvnent} = zeE/(k_B T/D)$.
- The particle current density becomes $j_s = -D\frac{dC}{dx} + Cj_v + CzeE\frac{D}{k_B T}$.

DIFFUSION: POTENTIALS AND TRANSPORT

- We consider the case where there is no bulk solution flow, the Nernst-Planck equation:
- $j_s = -D\frac{dC}{dx} + CzeE\frac{D}{k_BT}.$
- The physics here to intuitively understand is that diffusion always occurs towards the region of lower concentration, while for positive charge the V_{solut}term is in the direction of the electric field E.
- Our initial conditions are current density in bulk solution with $x = 0, v(x) = 0; x = L, v(x) = v$.
- Then, $j = -\frac{z^2e^2DCS}{k_BTL}\frac{v}{S} = -\frac{G(C)}{S}v.$
- This can be rewritten in terms of the conductivity, as defined from $G = \sigma S/L = 1/R = S/\rho L$
- $\sigma = \frac{1}{\rho} = \frac{z^2e^2DC}{k_BT}$

DIFFUSION: POTENTIALS AND TRANSPORT

- Some ion conductivities

- $$\sigma = \frac{1}{\rho} = \frac{z^2 e^2 DC}{k_B T}$$

Table 9.4 Conductivities of ions at various concentrations at 25°C, calculated using Eq. 9.39. Diffusion constants for each ion are from Hille (2001, p. 317). Concentrations are typical of mammalian nerve and are from Hille (2001, p. 17). The conductivities of each species add, and $\rho = 1/\sigma$. Larger ions with very small diffusion constants make the solutions electrically neutral

	D ($m^2 s^{-1}$)	C (mmol l^{-1})	σ (S m^{-1})	ρ (Ω m)
		Extracellular squid axon		
Na	1.33×10^{-9}	145	0.723	
K	1.96×10^{-9}	4	0.029	
Cl	2.03×10^{-9}	123	0.936	
			1.688	0.592
		Intracellular squid axon		
Na	1.33×10^{-9}	12	0.060	
K	1.96×10^{-9}	155	1.139	
Cl	2.03×10^{-9}	4.2	0.032	
			1.231	0.812

DIFFUSION: POTENTIALS AND TRANSPORT

- Some ion conductivities
- $\sigma = \dfrac{1}{\rho} = \dfrac{z^2 e^2 DC}{k_B T}$

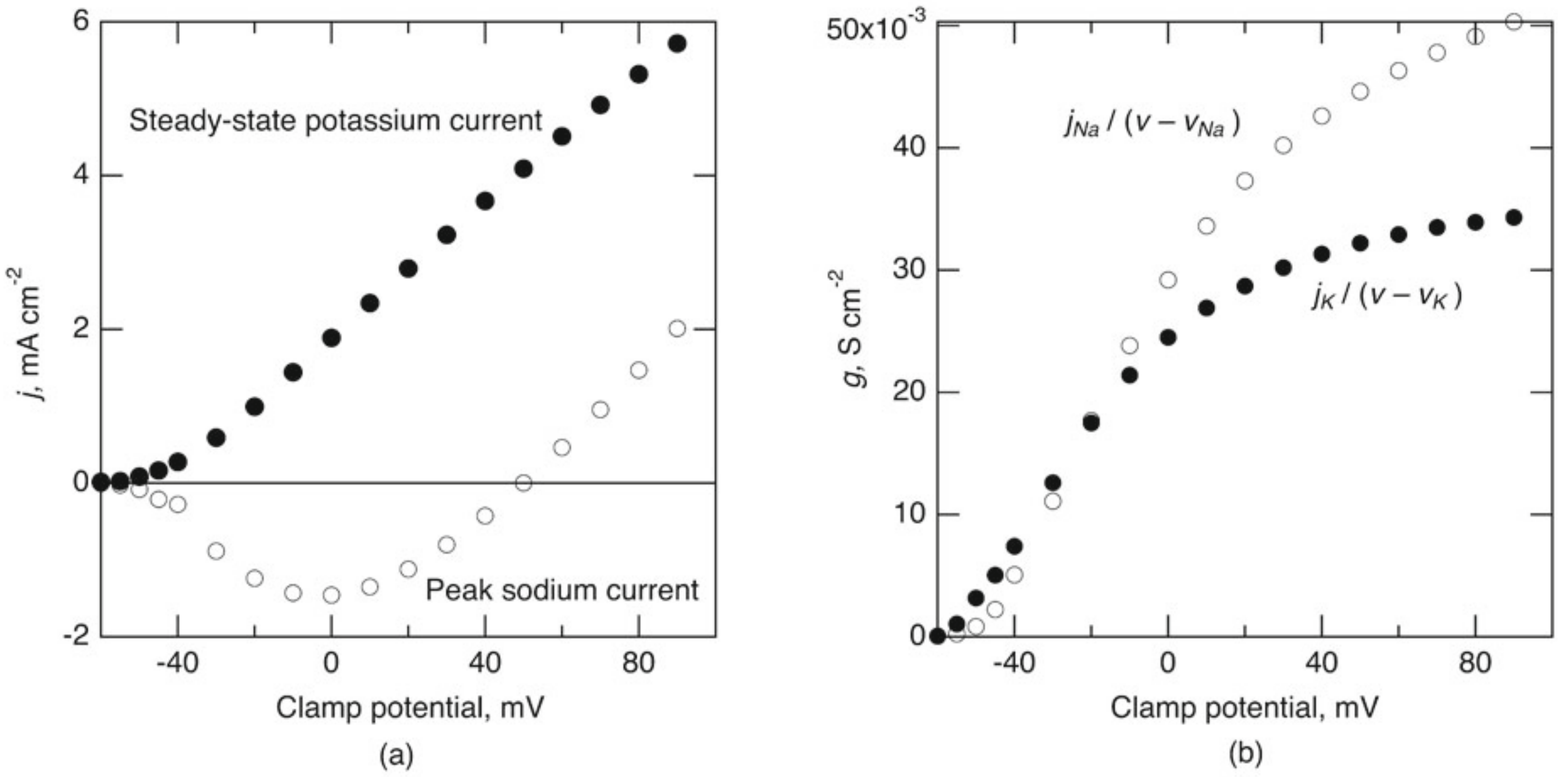
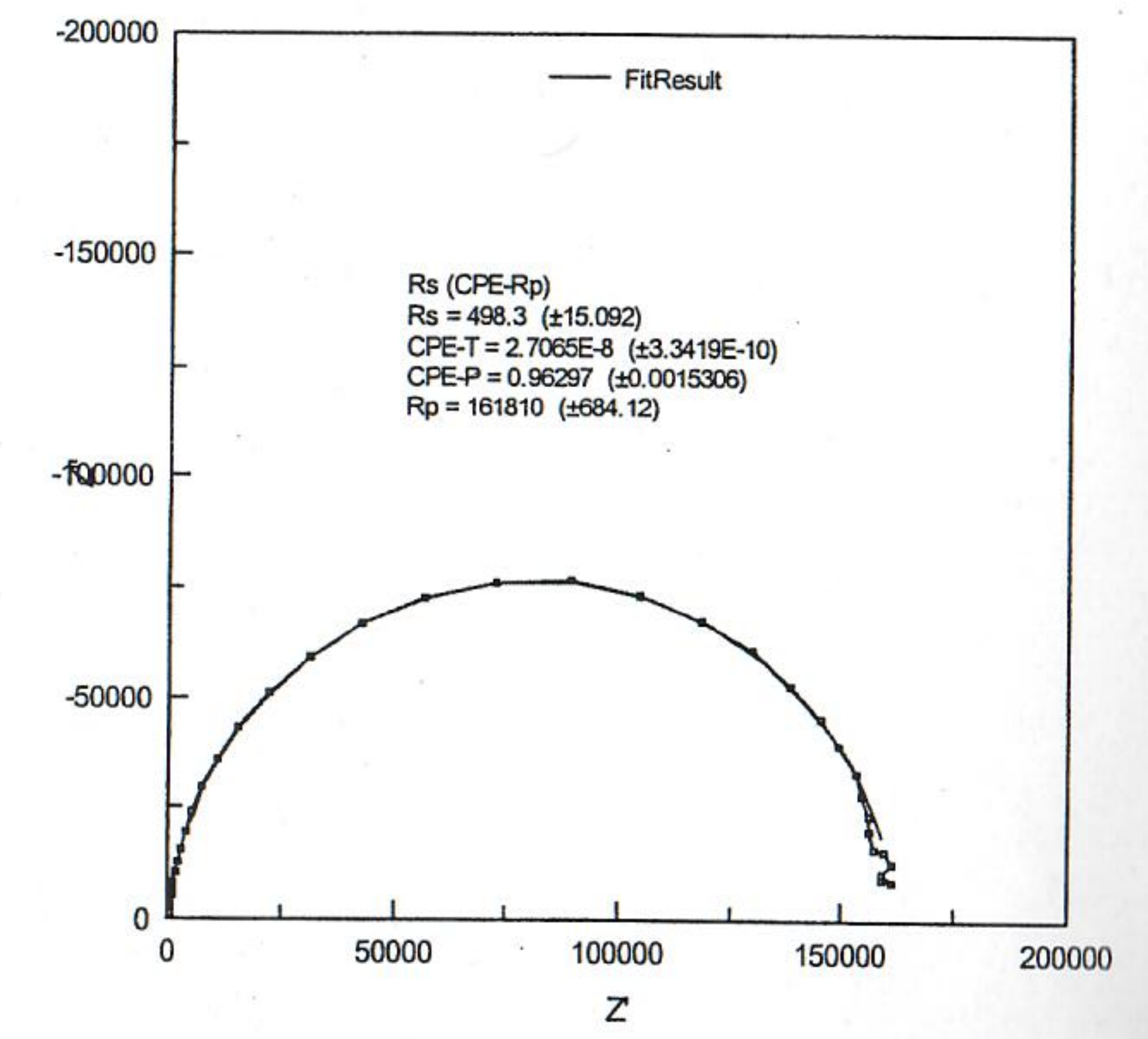


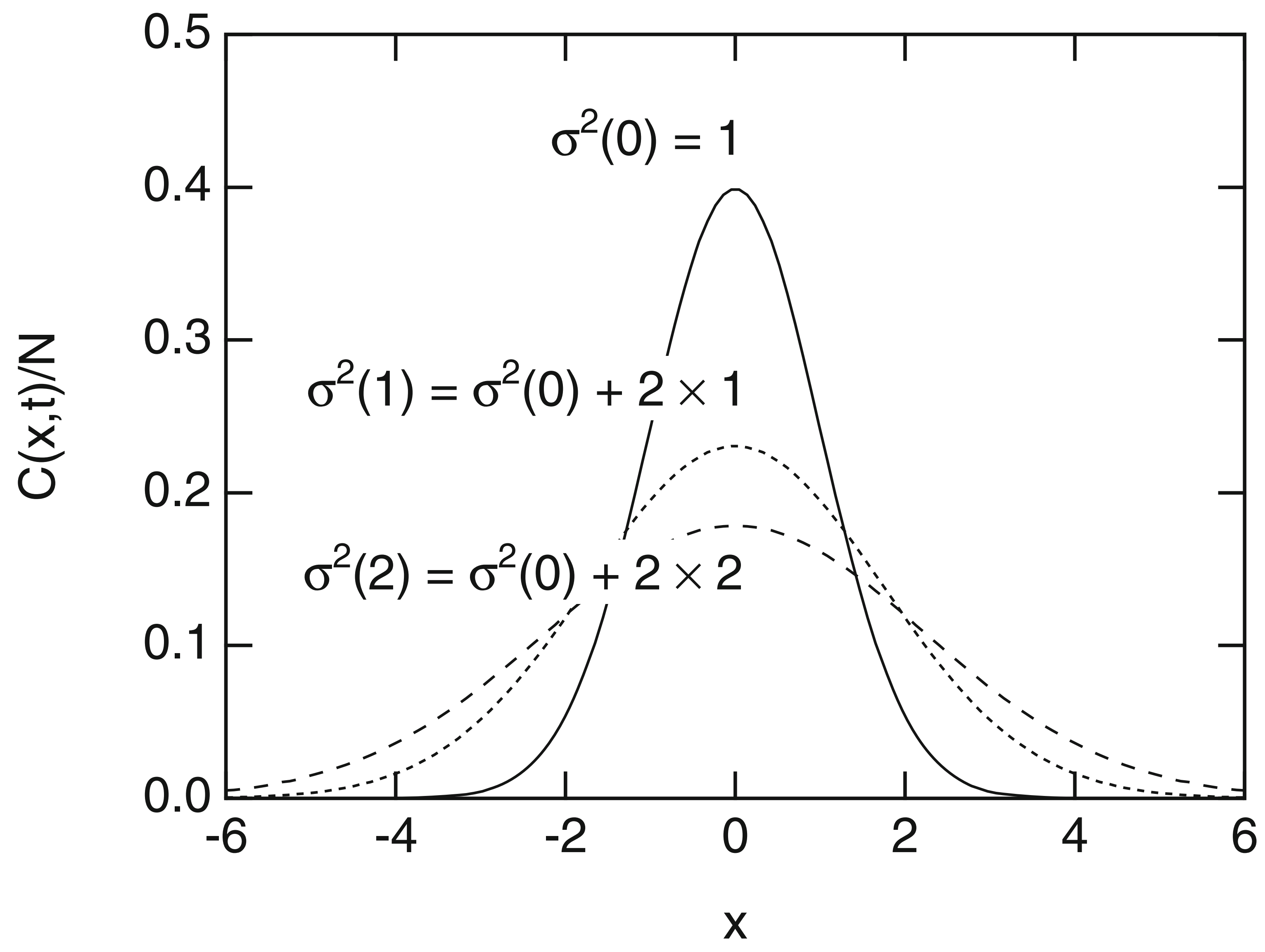
Fig. 9.12 Steady-state potassium current and peak sodium current for a squid axon subject to a voltage clamp vs. the transmembrane potential during the clamp. These are not real data, but were generated using the Hodgkin–Huxley model. **a** Current density. **b** Current density divided by the difference between the potential and the Nernst potential, to give the conductance per unit area. (see Eq. 6.61)

DIELECTRIC SPECTROSCOPY (?)

- From impedance spectroscopy, characterization of organic material can be obtained. An example is that of human blood.
- The basic model is a distributed element or constant phase element model with a capacitor like element with a reactance of the form
- $Z_T = \dfrac{1}{(j\omega)^P C_T} = R_s + \dfrac{R_P}{1 + R_P C_T (j\omega)^P}$ with $j = -1, \omega = 2\pi f, f$ the electric current frequency.



DIFFUSION: POTENTIALS AND TRANSPORT



Spreading of particles by diffusion assuming D = 1

- There is of course also a time dependence associated with understanding of Fick's law, the 2nd law.
- $$-\frac{\partial C}{\partial t} = D(\nabla^2 C) = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right)$$
- The solution in 1D for the concentration in then
- $$C(x,t) = \frac{N}{\sqrt{2\pi\sigma(t)}} e^{-x^2/2\sigma^2(t)}$$
- We can check this by plugging in this formula and equating both sides.

DIFFUSION: POTENTIALS AND TRANSPORT

- A general solution, for the concentration can be estimated (we'll just show this slide) because we assume that a particle does not stay put and acquires a mean square velocity $3k_BT/m$ so that

- $$C(x,t)dx = \frac{1}{\sqrt{4\pi Dt}}\int_{-\infty}^{\infty} C(\xi,0)e^{(-x-\xi)^2/4Dt}d\xi$$

- If the initial concentration, C_0 is a constant,

-

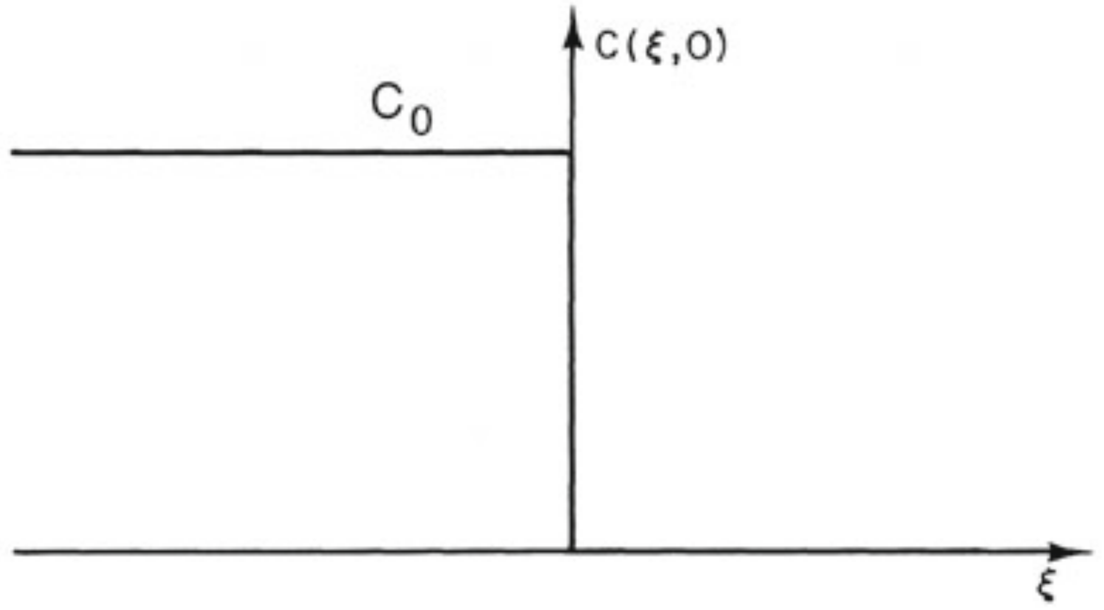


Fig. 4.20 The initial concentration is constant to the left of the origin and zero to the right of the origin

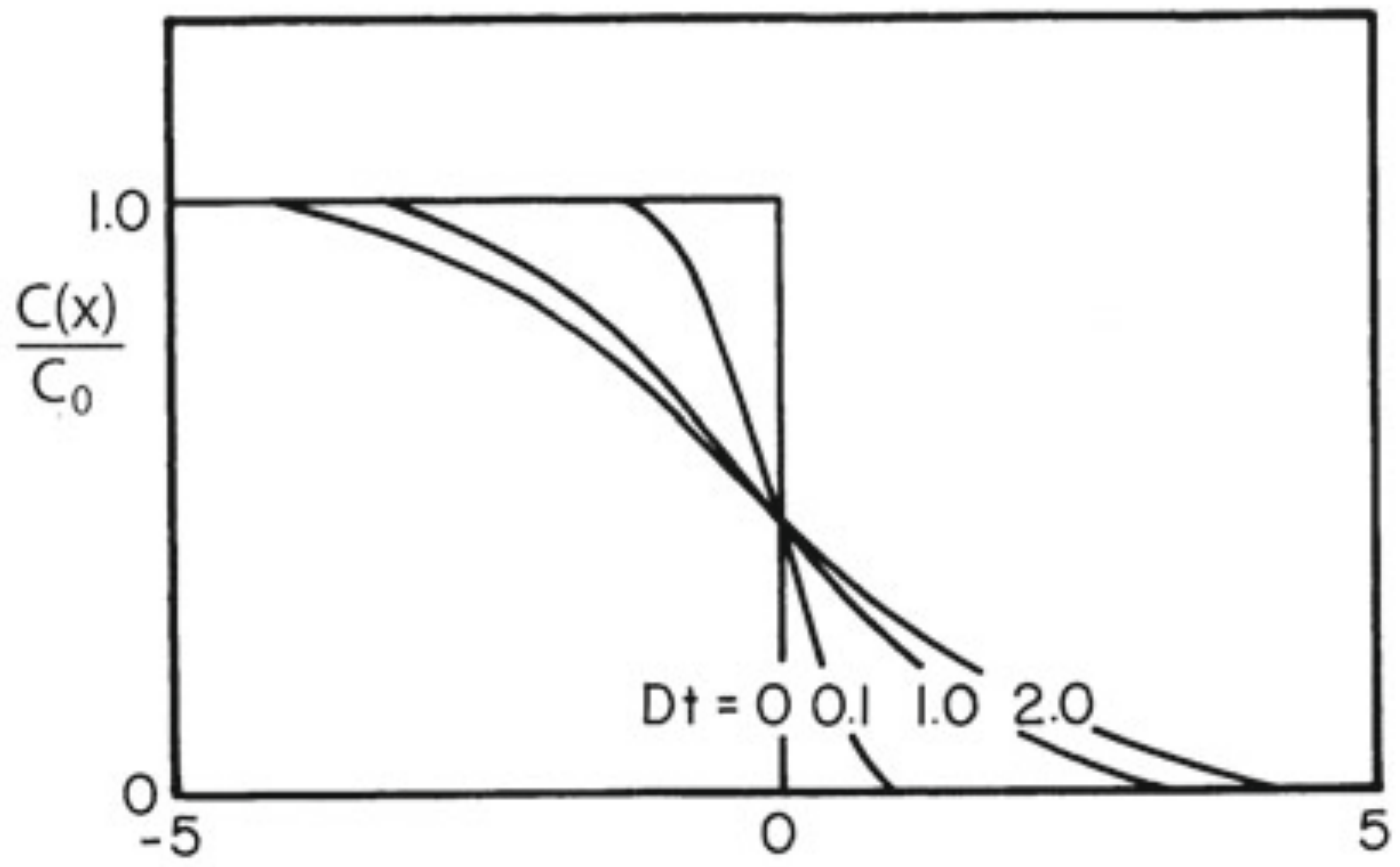


Fig. 4.22 The spread of an initially sharp boundary due to diffusion